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The house size effect and the referendum paradox in U.S. presidential elections

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ABSTRACT

Barthélémy et al. (2014), extending the work of Neubauer and Zeitlin (2003), show that some U.S. presidential elections are subject to a 'House size effect' in that the winner of the election, i.e., the candidate who wins a majority of electoral votes, depends on the size of the House of Representatives. The conditions for the effect relate to the number of 'Senate' versus 'House' electoral votes won by each candidate, but the relationship is not straightforward due to 'locally chaotic' effects in the apportionment of House seats among the states as House size changes. Clearly a Presidential election that is subject to the House size effect exhibits the referendum paradox, i.e., the electoral vote winner is the popular vote loser, for some House sizes but not for others.

1. Introduction

The U.S. Constitution fixes some features of the Electoral College system for electing Presidents, including the stipulations that each state has electoral voters equal in number to its total representation in Congress and that each state has two Senators. However, the Constitution does not fix the size of the House of Representatives, which can be changed by statute.

Some years ago, Neubauer and Zeitlin (2003) observed that, if the ratio of House seats to population that existed in 1940 had been maintained in 1990 (the census and apportionment that governed the 2000 election), the House would have had about 830 members. Moreover, Neubauer and Zeitlin calculated that, given a House of 830 members (and the existing Hill–Huntington apportionment method), Gore would have won the 2000 election with 471 electoral votes to 463 for Bush. More generally, they show that Bush would have won the election with any House size through 490, Gore would have won with any House size of 598 or greater (except for an electoral vote tie with a House of 655 members), and outcomes for House sizes between 491 and 597 would have fluctuated in a chaotic fashion between narrow Bush victories, narrow Gore victories, and perfect ties.

Using what they call 'representation graphs,' Barthélémy et al. (2014) have recently expanded Neubauer and Zeitlin's analysis in several ways: first, they examine additional presidential elections; second, they consider the effects of additional apportionment methods; third, they allow for other Senate sizes and House representation floors (the Constitution guarantees every state one House seat); and fourth, they consider the phenomenon of the 'referendum paradox' (or 'election inversion,' in the language of Miller, 2012) that was a notable feature of the 2000 election, i.e., the fact that Bush won a majority of electoral votes while Gore won a plurality of popular votes.

Somewhat surprisingly, neither Neubauer and Zeitlin nor Barthélémy et al. in the original version of their paper sought to identify the logical conditions that determine whether what we may call the 'House size effect,' as was present in the 2000 election, can arise. The purpose of this note is to identify these conditions and to draw out their implications.

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Table 1
Selected historical presidential elections.

(1) Category of election	(2) Election	(3) PV winner	(4) States carried		(5) Senate EV		(6) House EV (actual)		(7) Pop. %[a]		(8) House EV (perf. apport.)		(9) Crossover house size (perf. app.)
			Dem	Rep	Dem	Rep	Dem	Rep	Dem	Rep	Dem	Rep	
Clearly subject to house size effect	2000[b]	Dem	21	30	42	60	225	211	51.68	48.32	225.34	210.66	534.64
	1976	Dem	24	27	48	54	249	187	57.37	42.63	250.14	185.86	40.69
	1960[d]	Dem	23	26	46	52	263	168	60.24	38.31	263.24	167.43	29.30
	1916	Dem	30	18	60	36	216	219	49.91	50.09	217.60	218.40	12938.84
	1876[b]	Dem	17	21	34	42	150	143	51.80	48.20	151.77	141.23	222.48
	1860[e,b]	Dem	18	15	36	30	98	139	40.98	59.02	97.11	139.89	33.25
Partial effect	1880	Rep	19	19	38	38	117	176	40.32	59.64	118.15	174.85	0
	1848	Whig	15	15	30	30	97	133	42.52	57.48	97.79	132.21	0
Possibly subject (but not)	1896	Rep	22	23	44	46	131	226	36.22	63.78	129.29	227.71	–
	1888[c]	Dem	18	20	36	40	132	193	41.04	58.96	133.39	191.61	–
	1884	Dem	20	18	40	36	179	146	52.51	47.49	179.86	145.14	–

[a] Percent of apportionment population in states carried.
[b] Referendum Paradox with actual House size (not 'entrenched').
[c] Referendum Paradox with any House size ('entrenched').
[d] Kennedy is credited with carrying Alabama and winning all of its electoral votes; an unpledged elector slate carried Mississippi (8 electoral votes).
[e] Counterfactual two-candidate election: Lincoln vs. united opposition.

Source: Popular and electoral vote data are available from many sources; the most authoritative may be *Guide to U.S. Elections*, 5th ed., CQ Press, 2005. Apportionment populations are from Balinski and Young (2001), Appendix B; their figures come from the Census Bureau and other sources noted on p. 157. When a state was admitted since the last decennial census and apportionment, its territorial population in the previous census is used, with the exception of Minnesota and Oregon in 1860, for which their (state) population in the 1860 census were used. (Minnesota and Oregon were admitted as states in 1858 and 1859; both were virtually unpopulated in 1850.) Territorial populations are taken from http://en.wikipedia.org/wiki/ List_of_U.S._states_by_historical_population, which in turn cites the U.S. Census.

2. Senate versus house electoral votes and the house size effect

In the language of Madison's Federalist 39, the apportionment of electoral votes is a compromise between the 'federal' principle of state equality embodied in the Senate and the 'national' principle of state representation according to population embodied in the House of Representatives. The ratio of House to Senate size determines the weight given to the 'national' versus 'federal' principles in the Electoral College. Given a fixed number of states, the size of the House determines this ratio; an increase in the size of the House makes the Electoral College more 'national' and the allocation of electoral votes among the states more proportional to their populations.

For purposes of this analysis, we deem a presidential election to be a two-candidate affair, specified by the popular vote for each candidate in each state. It is assumed that the candidate who wins the popular vote of a state wins all of its electoral votes and the District of Columbia is treated as if it were a state. A candidate is said to win two 'Senate' electoral votes for each state he carries, the remainder being 'House' electoral votes.

Typically the federal and national principles give the same verdict — that is, one candidate carries a majority of states and therefore wins a majority of Senate electoral votes and the same candidate wins a majority of House electoral votes. But sometimes the two principles give conflicting verdicts. For example, in the 2000 election Bush carried 30 states and Gore carried 21, giving Bush a margin of 18 with respect to Senate electoral votes, while Gore won 225 House electoral votes to 211 for Bush, giving Gore a margin of 14, so Bush won by an overall margin of four electoral votes.[1] However, if the House size had been larger, Gore's House electoral vote margin would have been increased in about the same proportion and, with a sufficiently larger House, Gore would have won an overall electoral vote majority.

We say that a presidential election is subject to the *House size effect* if the winner of the election, i.e., the candidate who wins a majority of electoral votes, depends on the size of the House. The previous discussion appears to support the following proposition.

Proposition A. *A presidential election is subject the House size effect if and only if*:

(1A) *one candidate* (*say A*) *wins a majority of the Senate electoral votes*, *and*
(2A) *the other candidate* (*say B*) *wins a majority of the House electoral votes*.

As a summary of the historical record of presidential elections since 1828, Proposition A in fact holds up. The top rows of Table 1 show the six presidential elections that meet conditions (1A) and (2A); all these elections are subject to the House size effect and no others are. We will discuss these elections in more detail in Section 5.

But as a theoretical proposition, Proposition A does not (quite) hold up. This is because the apportionment of House seats is an unavoidably quirky matter, given that

[1] Gore lost one electoral vote to a faithless elector. The number of House electoral votes is one greater than the actual House size (i.e., 436 rather than 435) because DC is deemed to be a state.