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# A note on the consistent aggregation of nested logit demand functions

Olga Ivanova *

*Department of Applied Economics, Free University of Brussels, Avenue F Roosevelt 50 C. P. 140, 1050 Brussels, Belgium*

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**Abstract**

The present paper derives a set of rules allowing for the consistent aggregation of nested logit travel demand functions across origin and destination zones. Presented aggregation rules are derived for the case when the mode choice is performed conditional on destination choice. The derivation is based on the principles of consistency between aggregate and disaggregate travel demand models introduced by Sweet as well as upon the sampling theory.


*Keywords:* Nested logit model; Aggregation over travel zones

## 1. Introduction

In the recent years increasing attention has been paid to the development of sketch planning transportation models. These models are quite easy to construct and run in terms of both time

* Tel.: +32 2 650 3988; fax: +32 2 650 4137.
*E-mail address:* olga.ivanova@ecomod.net

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and financial resources. They may be used to look upon a large number of different combinations of transport policies with the best ones being analysed further using elaborate transport models. The use of sketch planning models allows one to save a lot of time and increase the probability that the best combination of transport related policies is implemented. It is quite important however that the results of small and large models correspond each other.

The present paper provides a method of constructing travel demand functions of a sketch-planning model, which are consistent with an underlying detailed nested logit transport demands. The present paper demonstrates how nested logit travel demand functions defined for an elaborated zone system may be consistently aggregated to match a simplified one. Aggregation rules are derived for the case when mode choice is performed at the lower level and are based on the principles of consistency introduced by Sweet (1997) as well as upon the sampling theory. This method also may be used in cases when a choice set is quite large and a compact model would be more tractable.

Sweet (1997) postulates two general conditions necessary for the consistent aggregation of utilities associated with choice alternatives. These conditions secure the consistency between choice model predictions at the aggregate and disaggregate levels. His first condition states that if a number of alternatives $i$ with a common systematic utility value are aggregated into a single composite alternative $I$, then that common value represents the systematic utility of the composite alternative $i \in I$. His second condition states that the choice probability of the composite alternative is the sum of probabilities of its respective components. The first condition specifies fixed demand requirement, while the second one ensures model consistency. He further implements the two conditions in order to derive formulas for aggregating logit travel demand functions across choice alternatives i.e. destination zones. The aggregation rules proposed in Sweet's paper have been used at MVA in the London Transportation Studies (LTS) for the construction of travel demand model (Department of Transport, UK, 1995).

The present paper extends Sweet (1997) by deriving formulas for the case of nested logit demand and aggregation across both origin and destination zones. In order to aggregate between destination zones Sweet's second condition is applied. Relationships between choice probabilities of aggregate and dissagregate alternatives (destination zones) are derived and represented in terms of their utility components. By substituting utility components of dissagregate alternatives by their common value, according to the Sweet's first condition, utility components of aggregate alternatives are derived.

The sampling theory is applied for aggregation over origin zones. In particular, I make use of the property that a share of individuals with certain characteristics in the total population is equal to a weighted sum of shares of such individuals in different population groups, with used weights being equal to shares of these groups in the total population. Relationships between choice probabilities of aggregate and disaggregate origin zones expressed in terms of their utility components are derived based on this sampling rule. By substituting utility components of dissagregate origin zones by their common value, according to the Sweet's first condition, utility components of aggregate origin zones are derived.

Structure of the paper is described as follows. Section 2 introduces some concepts and the notation used later in the paper. Section 3 provides a complete mathematical derivation of aggregation rules. Section 4 concludes the paper.