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Some properties and construction of multiwavelets related to different symmetric centers

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Abstract

In this paper we are interested in discuss the symmetry property and construction of an *m*-band compactly supported orthonormal multiwavelets related to the filters with different symmetric centers. With the development of the several equivalent conditions on this type of symmetry in terms of filter sequences and polyphase matrices, we derive several necessary constraints on the number of symmetric filters of the system, which is crucial for the construction of multiwavelets associated with given multiscaling functions with different symmetry centers. Then, we show how to construct multiwavelets with desired symmetric property by matrix extensions. Finally, to illustrate our proposed general scheme, we give two examples in this paper.

MSC: 42C40; 65T60

Keywords: Different symmetric centers; Orthogonality; Multiscaling function; Matrix extension; Multiwavelet

1. Introduction

In the theory and applications of wavelets, certain properties are always desirable. Symmetry, for example, is a crucial property in image processing. For symmetric filter, symmetric extensions transform of the finite length signals can be carried out, which improve the rate-distortion performance in image

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compression [1,2]. Recently, multiwavelets and their application have been much attention [3–13] due to the fact that it can simultaneously possess such as compactly supported, orthogonality and symmetry property, which is not possible for any real-valued 2-band scalar wavelets [14]. The GHM multiwavelet which was constructed by Geronimo et al. [5] using fractal interpolation is the first one to combine symmetry, orthogonality and short support, at the same time. Since then, many have worked on the design of symmetric/antisymmetric orthonormal multiwavelets [8,11,12,15,16]. Most deals with 2-band multiwavelet systems, a few authors have studied m-band multiwavelets, for example, the parameterization of m-channel orthogonal multifilter banks was developed in [17]. However, we note that these research mainly concerned multiscaling functions with all components having the same symmetric center. On the other hand, for a multifilter bank, since the input are vector signals, it is also required that the corresponding multiwavelets be balanced [18,19]. It was shown in [20] that the components of multiscaling functions must have the different symmetric center in order to construct symmetric and balanced multiwavelets. The purpose of this paper is investigate m-band orthonormal multiwavelet systems associated with such type symmetric property, where $m > 1$ is an arbitrary integer. To this end, we are needed to analysis symmetric property of orthogonal filters for such types of orthonormal multiscaling functions and multiwavelets. We called the corresponding filter to be a filter with different symmetric centers.

Matrix extension is one of the important approach for the constructing multiwavelets associated with a given multiscaling functions. The paraunitary matrix extension problems corresponding to the construction of orthonormal multiwavelets have been well studied [10]. Recently, Jiang studied symmetric paraunitary matrix extensions associated with 2-band compact supported orthonormal multiwavelets and shown that such matrix extension is always solvable [16]. In our preceding paper [23], we dealt mainly with the construction of m-band orthonormal multiwavelets corresponding to multiscaling functions that all the components having the same symmetric center. In this paper, we will concerned with orthonormal multiwavelets related to different symmetric center filter. This work is very interesting and more complicated. Several important results are presented but having some new feature. We systematically investigate the some properties of m-band multiwavelets system related to different symmetric center filter. In fact, we present the characterization of multiscaling functions and multiwavelets with the type symmetry in terms of matrix filter and polyphase matrix. Exploiting this property, we derived several necessary constraints on the number of symmetric/antisymmetric filters, which help to design multiwavelets with the desired symmetric property. It follows that we discuss the problem of matrix extension associated with multiscaling functions with different symmetric center for even m and some odd m. This lead to an practical algorithm for the construction multiwavelets with some symmetric property by matrix extension. This can be applied in the construction of multiwavelets if the multiscaling functions and low-pass filter are known. Finally, some examples are given by applying the proposed scheme.

Throughout this paper: let A^*, A^{T} and $\mathrm{Tr}(A)$ denote conjugate transpose, transpose and the trace of A, respectively. I_n denotes the $n \times n$ identity matrix, $0_{s,j}$ denote $s \times j$ zero matrix, for convenience, we omit the subscript $s \times j$ when it does not cause any confusion. Let $\mathrm{O}(n)$ denote the set of all $n \times n$ real orthogonal matrices. For two matrices $B = (b_{ij})$ and $C = (c_{ij})$, let $B \otimes C = (b_{ij}C)$ denote the Kronecker product of B, C. For $n \times n$ antidiagonal matrix J_n, we assumption: $J_0 \otimes A = 1$. δ_k is the Dirac sequence such that $\delta_0 = 1$ and $k \in Z\backslash\{0\}$ for all $\delta_k = 0$. $\lfloor\cdot\rfloor$ is the floor function. In this paper, we consider causal matrix sequence with real finite impulse response (FIR), such sequences can be identified with Laurent polynomial defined by $H(z) = \sum_{k=0}^{L} h(k)z^{-k}$, where 0 and L are the smallest and largest indices that $h(k)$ is nonzero, respectively.