Available online at www.sciencedirect.com
SCIENCE DIRECT®
Mathematics and Computers in Simulation 70 (2005) 133–148
MATHEMATICS AND COMPUTERS IN SIMULATION
www.elsevier.com/locate/matcom

Global stability analysis of a class of delayed cellular neural networks

Chuangxia Huang, Lihong Huang*, Zhaohui Yuan

College of Mathematics and Econometrics, Hunan University, Changsha, Hunan 410082, PR China

Received 11 November 2004; accepted 1 June 2005
Available online 11 August 2005

Abstract

Employing Brouwer's fixed point theorem, matrix theory, a continuation theorem of the coincidence degree and inequality analysis, the authors study further global exponential stability and the existence of periodic solutions of a class of cellular neural networks with delays (DCNNs) in this paper. A family of sufficient conditions is given for checking global exponential stability and the existence of periodic solutions of DCNNs. The results extend and improve the earlier publications.

Keywords: Cellular neural network; Exponential stability; Periodic solution; Lyapunov functional; Coincidence degree

1. Introduction

A cellular neural networks (CNN) is formed by many units called cells, the structure of the CNN is similar to that found in cellular automata, namely, any cell in a cellular neural network is connected only to its neighbor cells. A cell contains linear and nonlinear circuit elements, such as capacitors, resistors, controlled sources and so on. The circuit diagram and connection pattern modelling a CNN can be found in Refs. [1,2]. Nowadays, cellular neural networks are widely used in applications to signal and image processing, associative memories, pattern classification (see for instance [2–5]). Processing of

* Corresponding author. Tel.: +86 7318823056; fax: +86 7318823056.
E-mail address: lhhuang@hnu.cn (L. Huang); huangchuangxia@sina.com.cn (C. Huang).

0378-4754/$30.00
doi:10.1016/j.matcom.2005.06.001

moving images requires the introduction of delay in the signals transmitted among the cells [3]. There exist some results of stability for CNNs and delayed CNNs (DCNNs), we refer to Refs. [1–17]. To the best of our knowledge, few authors [6–10,13,14] have considered global exponential stability and periodic solutions for the DCNNs with unbounded activation functions. However, in some applications, one is required to use unbounded activation functions. For example, when neural networks are designed for solving optimization problems in the presence of constraints (linear, quadratic, or more general programming problems), unbounded activations modelled by diode-like exponential-type functions are needed to impose constraints satisfaction. The extension of the quoted results to the unbounded case is not straightforward. Different from the bounded case where the existence of an equilibrium point is always guaranteed [15], for unbounded activations, it may happen that there is no equilibrium point (see [16]). When considering the widely employed piecewise-linear activations, making it of interest to drop the assumptions of strict increase and continuous first derivative for the activation. Morita [17] showed that the absolute capacity of an associative memory model can be remarkably improved by replacing the usual sigmoid activation functions with nonmonotonic activation functions. Therefore, it seems that for some purposes, nonmonotonic (and not necessarily smooth) functions might be better candidates for neuron activation in designing and implementing an artificial neural network. In this paper, without assuming the boundedness, monotonicity, and differentiability of activation functions, we investigate further a class of DCNNs, which can be described by delayed differential equations (namely, functional differential equations), analyze further problems of global exponential stability and the existence of periodic solutions of DCNNs and derive some new and simple sufficient conditions, which are weaker than those in existing works.

In the following, we consider the global exponential stability and the existence of periodic solutions of DCNNs model described by differential equations with delays

$$\dot{x}_i(t) = -c_i(t)x_i(t) + \sum_{j=1}^{n} a_{ij}(t)f_j(x_j(t)) + \sum_{j=1}^{n} b_{ij}(t)f_j(x_j(t-\tau_{ij}(t))) + I_i(t), \tag{1.1}$$

$i = 1, 2, \ldots, n,$ in which n corresponds to the number of units in a neural network.

$x_i(t)$ denotes the potential (or voltage) of the cell i at time t; $f_i(\cdot)$ denotes a nonlinear output function; $I_i(t)$ denotes the ith component of an external input source introduced from outside the network to the cell i at time t; $c_i(t)$ denotes the rate with which the cell i resets its potential to the resting state when isolated from other cells and inputs at time t; $a_{ij}(t)$ and $b_{ij}(t)$ denotes the strengths of connectivity between the cell i and j at time t respectively; $\tau_{ij}(t)$ corresponds to the time delay required in processing and transmitting a signal from the jth cell to the ith cell at time t.

Obviously, model (1.1) is the most popular and typical neural network model. Some other models, such as continuous BAM (bidirectional associative memory) networks and Hopfield-type neural networks, are special cases of the network model (1.1) (see for instance [11,18]).

As pointed out by Gopaldamy and Sariyasa [9,10], it would be of great interest to study the neural networks in periodic environments. Motivated by this, in studying (1.1), we assume the following hypothesis.

Hypothesis 1.1. (H_1) $c_i, \tau_{ij} \in C(R, (0, \infty))$, $a_{ij}, b_{ij}, I_i \in C(R, R)$ are periodic functions with a common period $\omega(> 0)$, and $f_i \in C(R, R)$, $i, j = 1, 2, \ldots, n$.

Hypothesis 1.2. (H_2) $|f_j(u)| \leq p_j|u| + q_j$ for all $u \in R$, $j = 1, 2, \ldots, n$, where p_j, q_j are nonnegative constants.