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Domain decomposition for generalized unilateral semi-coercive contact problem with given friction in elasticity

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Abstract

A non-overlapping domain decomposition is applied to a multibody unilateral contact problem with given friction (Tresca's model). Approximations are proposed on the basis of the primary variational formulation (in terms of displacements) and linear finite elements. For the discretized problem we employ the concept of local Schur complements, grouping every two subdomains which share a contact area. The proposed algorithm of successive approximations can be recommended for "short" contacts only, since the contact areas are not divided by interfaces. The numerical examples show the practical efficiency of the algorithm.

Keywords: Domain decomposition; Unilateral contact; Tresca‘s friction model; Formulation in displacements; Linear finite elements

1. Introduction

In mechanics, geomechanics and biomechanics as well as technological practice there are problems whose investigations lead to solving model problems based on variational formulations. Such problems are described frequently by variational inequalities. Variational inequalities physically describe the principle of virtual work in its inequality form.

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Domain decomposition for contact problems has been applied by many authors. An augmented Lagrangian method was used by Dostál et al. [1]. The scalability of the algorithm has been studied by Schöberl [2], Dureisseix and Farhat [3] and Dostál and Horák [4]. Boundary element technique was employed by Kosior et al. [5]. For related papers, we refer to Luo Ping and Liang Guoping [6] and the literature therein.

In the present paper we will deal with numerical solution of a generalized semi-coercive contact problem with the given friction arising in static and quasi-static linear elasticity, for the case that several bodies of arbitrary shapes are in mutual contacts and are loaded by external forces, by using the non-overlapping domain decomposition method. The problem will be formulated as the primary variational inequality problem (see [21]), i.e. in terms of displacements.

We follow the approach proposed by Le Tallec [7] and group every two subdomains which share a contact area into a single "nonlinear" subdomain (see a similar idea used by Barboteu et al. [8]).

Section 2 contains both classical and weak formulation of the problem. Conditions sufficient and necessary for the existence of a weak solution are given. Approximations by linear finite elements on triangulation are proposed in Section 3 together with error estimates. In Section 4 we introduce a nonoverlapping domain decomposition by proving the equivalence of the weak solution on the original domain with that on the interface and subdomains. For the discretized version we employ the concept of local Schur complements. The resulting nonlinear equation on the interface is solved by successive approximations. For the starting approximation we choose the solution of the linear problem, where the unilateral contact conditions are replaced by classical bilateral conditions without friction.

Sections 5 and 6 are devoted to the construction of suitable preconditioning matrices of Neumann–Neumann type. In Section 7 we study the convergence of successive approximations. A sufficient condition and an error estimate is deduced on the basis of contractive mappings theorem. Section 8 contains a numerical test solving a geomechanical model with two domains in contact.

Though the solution of the problem with given friction (Tresca's model) has little physical meaning itself, it can be plugged into an iterative process for the solution of a more realistic Coulomb friction model (see [9–11]).

Since we do not divide the contact areas by interfaces, the proposed algorithm can be recommended for "short" contact only. Such configurations occur e.g. in models of human joints – hips or knees (see [12,13]).

2. Model formulation

Let the investigated part of the elastic body occupy a union Ω of "s" bounded domains Ω^ι, $\iota = 1, \ldots, s$ in $\mathbb{R}^2$, with Lipschitz boundaries $\partial\Omega^\iota$. Let the boundary $\partial\Omega = \bigcup_{\iota=1}^{s} \partial\Omega^\iota$ consist of four disjoint parts, i.e. $\partial\Omega = \overline{\Gamma}_u \cup \overline{\Gamma}_\tau \cup \overline{\Gamma}_c \cup \overline{\Gamma}_o$. Let us denote

$$\Gamma_c^{kl} = \partial\Omega^k \cap \partial\Omega^l,\ k, l = 1, \ldots, s, k \neq l, \Gamma_c = \cup_{k,l}\Gamma_c^{kl},\ \Gamma_u = \cup_{\iota=1}^{s}\Gamma_u^\iota,$$

$$\Gamma_u^\iota = \Gamma_u \cap \partial\Omega^\iota, \Gamma_o^\iota = \Gamma_o \cap \partial\Omega^\iota, \Gamma_\tau = \cup_{\iota=1}^{s}\Gamma_\tau^\iota, \Gamma_\tau^\iota = \Gamma_\tau \cap \partial\Omega^\iota.$$

Assume that either

$$\text{meas}\,\Gamma_c^{kl} > 0 \text{ or } \Gamma_c^{kl} = \emptyset$$