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Discrete Optimization 2 (2005) 71–82
DISCRETE OPTIMIZATION
www.elsevier.com/locate/disopt

Batched bin packing☆

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Received 9 February 2004; received in revised form 19 November 2004; accepted 23 November 2004

Abstract

We introduce and study the batched bin packing problem (BBPP), a bin packing problem in which items become available for packing incrementally, one batch at a time. A batched algorithm must pack a batch before the next batch becomes known. A batch may contain several items; the special case when each batch consists of merely one item is the well-studied on-line bin packing problem. We obtain lower bounds for the asymptotic competitive ratio of any algorithm for the BBPP with two batches. We believe that our main lower bound is optimal and provide some support to this conjecture. We suggest studying BBPP and other batched problems.

Keywords: On-line algorithm; Lower bounds; Bin packing; Competitive ratio

1. Introduction, terminology and notation

In this paper, we study a variation of the classical bin packing problem (BPP). In BPP, we are given a set B of items $a_1, a_2, \ldots, a_n$ and a sequence of their sizes $(s_1, s_2, \ldots, s_n)$ (each size $s_i \in (0, 1]$) and are required to pack the items into a minimum number of unit-capacity bins. In other words, we need to partition B into a minimum number m of subsets $B_1, B_2, \ldots, B_m$ such that $\sum_{a_i \in B_j} s_i \leqslant 1$ for each $j = 1, 2, \ldots, m$. For recent surveys of BPP, see [3–5].

☆ Research was supported in part by the Leverhulme Trust. Research of Gutin was supported in part by the IST Programme of the European Community, under the PASCAL Network of Excellence, IST-2002-506778.

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1572-5286/$ - see front matter
doi:10.1016/j.disopt.2004.11.001

We introduce the *batched bin packing problem* (*BBPP*), a bin packing problem in which items become available for packing incrementally, one batch at a time. A *batched algorithm* must pack a batch before the next batch becomes known. A batch may contain several items; the special case when each batch consists of merely one item is the well-studied on-line bin packing problem. In the case of just one batch, we have the classical off-line BPP. In BBPP, an input sequence L is a *batched sequence*, namely, $L = (B_1, B_2, \ldots, B_k)$, where every B_j is a set of items and $B_i \cap B_j = \emptyset$ whenever $1 \leqslant i < j \leqslant k$.

BBPP may be of interest when, for example, items are delivered to a packing site by trucks, each truck containing several items. To the best of our knowledge, despite being a very natural generalization of BPP, BBPP has not been studied before. Somewhat similar yet different problems were studied under the collective name of semi-on-line problems (see, e.g., [3,4]).

In particular, Galambos and Woeginger [8] considered a version of the on-line bounded-space BPP where repacking of items within some active bins is allowed. For this problem, the lower bound ℓ_{LL} of Lee and Lee [13] ($\ell_{LL} \approx 1.69103$) for the competitive ratios of bounded-space approximation algorithms still applies. Galambos and Woeginger presented an algorithm that reaches the best possible competitive ratio matching ℓ_{LL} while using only three active bins. Algorithms with much more freedom to rearrange items were developed by Ivković and Lloyd [11,12]. Grove [9] considered a k-bounded lookahead algorithm, which delays packing an item till $k - 1$ next items has arrived, or the restricted capacity (of a warehouse) has been exceeded. Grove's algorithms achieves the optimal competitive ratio of ℓ_{LL} when the warehouse capacity is sufficiently large.

The on-line bin batching problem considered in a short note [17] is different from BBPP as it is an extension of the bin *covering* problem.

All batched sequences with exactly k batches (some of which may be empty) comprise a set, which we denote by $\mathscr{B}(k)$. For a given batched sequence L and batched algorithm A, let $A(L)$ be the number of bins required for L by algorithm A; let $\mathrm{OPT}(L)$ be the minimum number of bins needed to pack the items of L when they are all available at once (as in BBP). The *asymptotic competitive ratio* $R^{\infty}_{A,k}$ of A on $\mathscr{B}(k)$ is

$$\limsup_{N\to\infty} \max\left\{\frac{A(L)}{\mathrm{OPT}(L)} : L \in \mathscr{B}(k),\ \mathrm{OPT}(L) = N\right\}.$$

The asymptotic competitive ratio of a batched algorithm is defined similarly to the asymptotic competitive ratio of an on-line algorithm.

In this paper, we study lower bounds of $R^{\infty}_{A,2}$ for any batched algorithm A with inputs from $\mathscr{B}(2)$. We note that any additional assumptions, such as polynomiality, are not made about the algorithms that we study. In Section 2, we prove such a bound r in Theorem 1. We conjecture that the bound r is optimal. To formally support this conjecture we prove in Section 4 that the bound r is optimal for a wide family of batched sequences. In Section 3 we obtain lower bounds of $R^{\infty}_{A,2}$ for the restriction of $\mathscr{B}(2)$ to instances in which the number of item sizes is bounded (a natural constraint). Section 5 is devoted to open problems and suggestions for further research.

Yao [16] was the first to study lower bounds for the asymptotic competitive ratio of an on-line algorithm for BBP. He showed that such a bound is not smaller than 1.5. Brown [1] and Liang [14] independently improved Yao's result to 1.53635. This was further improved