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On the use of independent base-stock policies in assemble-to-order inventory systems with nonidentical lead times

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ABSTRACT

We consider the use of Independent Base Stock (IBS) replenishment policies in Assemble-to-Order (ATO) inventory systems. These policies are appealingly simple and widely used, but generally suboptimal for systems with non-identical lead times. We present an IBS policy and prove that its loss of optimality is limited by the ratio of the longest lead time to the shortest one. Our results suggest that IBS policies can work well for systems where differences between lead times are dominated by their lengths.

1. Introduction

Assemble-to-Order (ATO) manufacturing keeps inventory at the component level and assembles a final product only after the demand for it has arrived. Managing an ATO inventory system involves two control policies. The replenishment policy determines the ordering of components. The allocation policy distributes components to serve different demands. Minimizing the long-run average expected total inventory cost is a common objective that we consider in this paper.

Independent base-stock (IBS) policies are probably the most popular replenishment control schemes (e.g., see [9] for a review of the related literature). An IBS policy keeps each component's inventory position, defined as the difference between its on-hand plus on-order inventory and its backlog, at a constant level for all time. In systems with identical replenishment lead times, IBS policies can be asymptotically optimal in the long-lead time or high-volume asymptotic regimes [6]. They also perform well outside these regimes and are even optimal in some special cases [2].

IBS policies become inadequate in systems with non-identical lead times, even for those that contain only one product [7,11]. Nevertheless, finding a better alternative is also hard. Policies that vary inventory positions of some components to satisfy certain optimality criteria have been considered [3,5,7], but their developments have been so far limited to systems with special Bill of Materials (BOM). As a result, base stock policies, such as the ones that Ignore Simultaneous Stock-outs (ISS) of components [10], are still commonly employed in practice. This paper considers a family of IBS policies with different choices of base stock levels from existing schemes such as ISS. We show that *in many ATO systems with a general BOM and deterministic but different lead times, these policies keep the long-run average inventory cost close to its minimum*.

Previous studies have used stochastic programs (SP) to set a lower bound on the average inventory cost of ATO systems [5,6,10]. We start from a similar launching pad by formulating an SP and proving its optimal solution is below the inventory cost under any feasible policy. However, unlike the bound in [5], ours is based on a two-stage SP instead of a $K+1$ stage SP (where K is the number of different lead times). Unlike the bound in [6], ours applies to systems with any number of distinct lead times rather than one lead time only. Unlike the bound in [10], ours covers all feasible policies, not just IBS policies. Inventory control is optimal if it keeps the total expected cost at the lower bound for all time. This happens if, in the ATO system, the replenishment policy replicates the probability distribution of 'component balance' (which we define at the beginning of Section 4.2) of the first stage of the SP; and for any given realization of component balance, the allocation policy replicates the outcome of the second stage of the SP (a condition we refer to as *perfect allocation*).

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A perfect match of component balance between an ATO system *with nonidentical lead times* and its corresponding SP is generally impossible. Our key result, Theorem 2 presents an upper bound on the cost impact of the mismatch for systems under control of our IBS policy. The bound provides a useful performance assessment of the cost of following IBS policies in general. Moreover, for systems where variations of component lead times are dominated by their lengths, the bound implies that our policy entails little loss of optimality. Hence, the cost objective stays close to the achievable minimum under perfect allocation, which is attainable in some special cases [2,4,5].

While perfect allocation is unattainable in general ATO systems, a family of asymptotically-optimal allocation policies has been developed in [6]. These policies involve solving a certain linear program, whose parameters depend on the current state of the system (Section 5 provides more details). For systems with identical lead time, the percentage difference of the long-run average inventory cost between these policies and the perfect allocation converges to zero as the lead time grows [6]. We consider a joint use of these allocation policies and our IBS replenishment policy in systems with nonidentical lead times. We prove that the combination is asymptotically optimal as the lead times grow while their differences grow at a slower rate.

The rest of the paper is divided into four sections. We define the problem in Section 2, present our IBS policy in Section 3, derive the lower bound and carry out performance analysis in Section 4, and conclude the paper with a discussion on the implications of our results in Section 5.

2. Problem formulation

We develop our analysis for the continuous-review formulation. Our results also extend to the periodic-review model by analogous arguments.

We consider ATO systems with m products, n components, and a non-negative integer matrix A as its BOM. Here a_{ji} denotes the amount of component j $(1 \le j \le n)$ used by product i $(1 \le i \le m)$, so row j of A, denoted by $\mathbf{A}_j$, represents the usage of components j $(1 \le j \le n)$ by all products. There are K distinct lead times with $L_K > L_{K-1} \cdots > L_1 > 0$, and we use k_j $(k_j = 1, \ldots, K)$ to refer to the index of the lead time of component j. Let $n_0 = 0$ and n_k be the number of components with lead time L_k or shorter $(1 \le k \le K)$, so $n_K = n$. Let components be indexed according to the ascending order of their lead times, so $\{n_{k-1}+1, \ldots, n_k\}$ is the index set of components with lead time $L_k (1 \le k \le K)$. The usage of components with lead time L_k is given by A^k, which is a submatrix of A that contains rows $j = n_{k-1}+1, \ldots, n_k$ $(k = 1, \ldots, K)$ and columns $i = 1, \ldots, m$.

Demand arrives according to a compound Poisson process. The number of orders during the time interval $[0, t]$ is denoted by $\Lambda(t)$ $(t \ge 0)$ and $\underset{\sim}{\lambda} = \mathbf{E}[\Lambda(1)]$ is the order arrival rate. There is an associated i.i.d. sequence of random vectors that provide order sizes. A generic element of this sequence is denoted by $\mathbf{S} = (S_1, S_2, \ldots, S_m)$, where S_i is the order size of product i, $1 \le i \le m$. We assume that $\mathbf{S}$ has a finite second moment. (In Section 5, for one of our results we assume that $\mathbf{S}$ has a finite moment of order $2+\delta$ where δ is a positive value that can be arbitrarily small.) The total demand during $[0, t]$ is denoted by

$$\mathcal{D}(t) = (\mathcal{D}_1(t), \ldots, \mathcal{D}_m(t))', \quad t \ge 0,$$

with $\mathbf{E}[\mathcal{D}(1)] = \boldsymbol{\mu} < \infty$. Define

$$\mathbf{D}(t_1, t_2) = \mathcal{D}(t_2) - \mathcal{D}(t_1), \quad 0 \le t_1 \le t_2,$$

as the demand in the interval $(t_1, t_2]$. For $t \ge L_K$, let

$$\mathbf{D}^k(t) = \mathbf{D}(t - L_k, t - L_{k-1}), \quad 1 \le k \le K, \quad \text{and}$$
$$\bar{\mathbf{D}}^k(t) = \mathbf{D}(t - L_K, t - L_k), \quad 0 \le k \le K,$$

where $L_0 = 0$ (note that $\bar{\mathbf{D}}^0(t) = \mathbf{D}(t - L_K, t)$). Since the demand process is stationary, we can define random vectors $\mathbf{D}^k$ and $\bar{\mathbf{D}}^k$ such that

$$\mathbf{D}^k \overset{d}{=} \mathbf{D}^k(t) \quad (1 \le k \le K) \quad \text{and}$$
$$\bar{\mathbf{D}}^k \overset{d}{=} \bar{\mathbf{D}}^k(t) \quad (0 \le k \le K), t \ge L_K.$$

Let $\mathcal{R}_j(t)$ be the quantity of component j ordered from the supplier between time $-L_{k_j}$ and time t for $t \ge -L_{k_j}$ and define

$$\mathcal{R}(t) = (\mathcal{R}_1(t), \ldots, \mathcal{R}_n(t))', \quad t \ge 0.$$

Denote the total quantity of demand served during $[0, t]$ by

$$\mathcal{Z}(t) = (\mathcal{Z}_1(t), \ldots, \mathcal{Z}_m(t))', \quad t \ge 0.$$

Similarly, for $1 \le k \le K$, denote

$$\mathbf{R}^k(t) = \mathcal{R}(t) - \mathcal{R}(t - L_k) \quad \text{and} \quad \mathbf{Z}^k(t) = \mathcal{Z}(t) - \mathcal{Z}(t - L_k),$$
$$t \ge 0.$$

We consider a backlog model and denote the backlog levels at time t by $\mathbf{B}(t) = (B_1(t), \ldots, B_m(t))'$, and the per-unit backlog costs by $\mathbf{b} = (b_1, \ldots, b_m)'$. Denote the inventory levels of components with lead time L_k by $\mathbf{I}^k(t) = (I_{n_{k-1}+1}(t), \ldots, I_{n_k}(t))'$, and the corresponding unit inventory holding costs by $\mathbf{h}^k = (h_{n_{k-1}+1}, \ldots h_{n_k})'$ $(1 \le k \le K)$. Let

$$\mathbf{I}(t) = (I_1(t), \ldots, I_n(t))' \quad (t \ge 0) \quad \text{and} \quad \mathbf{h} = (h_1, \ldots, h_n)'$$

be concatenations of vectors $\mathbf{I}^k(t)$ $(1 \le k \le K)$ and $\mathbf{h}^k$ $(1 \le k \le K)$ respectively. Changes of backlog and inventory levels are governed by

$$\begin{aligned} &\mathbf{B}(t) = \mathbf{B}(t - L_k) + \mathbf{D}(t - L_k, t) - \mathbf{Z}^k(t), \quad 1 \le k \le K,\ t \ge L_K, \\ \text{and} \quad &\mathbf{I}^k(t) = \mathbf{I}^k(t - L_k) + \mathbf{R}^k(t - L_k) - A^k\mathbf{Z}^k(t), \quad 1 \le k \le K,\ t \ge L_K. \end{aligned} \tag{1}$$

The objective is to minimize the long-run average expected inventory cost

$$\mathcal{C} \equiv \limsup_{T \to \infty} \frac{1}{T} \int_{L_K}^{T+L_K} \mathcal{C}(t)dt, \tag{2}$$

where $\mathcal{C}(t) = \mathbf{b} \cdot \mathbf{E}[\mathbf{B}(t)] + \mathbf{h} \cdot \mathbf{E}[\mathbf{I}(t)] = \mathbf{b} \cdot \mathbf{E}[\mathbf{B}(t)]$

$$+ \sum_{k=1}^{K} \mathbf{h}^k \cdot \mathbf{E}[\mathbf{I}^k(t)].$$

In our discussion below, for any positive integer l, $\mathbb{R}_l$ and $\mathbb{R}_l^+$ respectively denote the sets of l-dimensional real vectors and non-negative real vectors.

3. Policy development

We define an IBS replenishment policy with the following base stock levels

$$\mathbf{Y}^k = \mathbf{y}^{k*} + (L_k - L_1)A^k\boldsymbol{\mu}, \quad 1 \le k \le K. \tag{3}$$

Here $\mathbf{y}^{k*} = (y^*_{n_{k-1}+1}, \ldots, y^*_{n_k})'$ $(1 \le k \le K)$ are subvectors of $\mathbf{y}^* = (y^*_1, \ldots, y^*_n)'$, the optimal solution of the following two-stage stochastic program (SP)

$$\tilde{\Phi} = \min_{\mathbf{y} \in \mathbb{R}_n} \{\mathbf{h} \cdot \mathbf{y} + \mathbf{E}[\tilde{\Phi}^0(\mathbf{y}, \mathbf{D}^1)]\} \tag{4}$$

$$\text{where } \tilde{\Phi}^0(\mathbf{y}, \mathbf{x}) = -\max_{\mathbf{z} \in \mathbb{R}_m} \{\mathbf{c} \cdot \mathbf{z} | \mathbf{z} \le \mathbf{x}, A\mathbf{z} \le \mathbf{y}\}, \tag{5}$$

and $\mathbf{c} = \mathbf{b} + A^T\mathbf{h}$. Our policy is a generalization of an IBS policy defined in [6]. In [6], all components have the same lead time L_1, and $\mathbf{y}^{k*}$ $(1 \le k \le K)$ are prescribed as base stock levels to serve demands occurring over a period of that length. These base