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Splitting versus setup trade-offs for scheduling to minimize weighted completion time

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Abstract

We study scheduling problems when jobs can be split and a setup is required before processing each part, to minimize the weighted sum of completion times. Using a simple splitting strategy and a reduction to an orders scheduling problem we derive a 2-approximation algorithm for the case with uniform weights and setups, improving upon previous work. We extend this idea to the general identical machine case and conclude by designing a constant factor approximation algorithm when machines are unrelated.

1. Introduction

Scheduling problems in which jobs are allowed to be split into many parts and be processed independently and concurrently on different machines naturally model situations in which jobs consist of a large number of small identical operations. When processing a job (or its small operations) the setup cost is negligible, while when switching to a different job a setup time is required, during which the machine cannot process or setup any other job (see Figure 1). This family of scheduling problems was introduced by Potts and Van Wassenhove [9] to study batching and lot-sizing integrated with scheduling decisions in manufacturing.

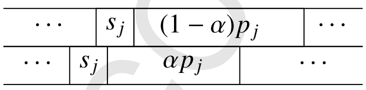

Figure 1: Example where a job is split in two pieces.

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In this paper we consider a stylized version of the model in which jobs can be arbitrarily split, introduced by Serafini [11] as a scheduling model for the textile industry. More specifically, our model considers a set M of m machines and a set J of n jobs under two different machine environments: unrelated and identical machines. In the unrelated machine setting, each job $j \in J$ has a machine dependent setup time s_{ij} and processing time p_{ij}, while for identical machines the processing and setup times are machine independent, so $p_{ij} = p_j$ and $s_{ij} = s_j$ for all $j \in J$ and $i \in M$. On the other hand, each job $j \in J$ is associated with a nonnegative weight w_j and the objective is to minimize the weighted sum of job completion times. Thus, the problems studied are denoted by $R|\text{split}|\sum w_jC_j$ (unrelated machines) and $P|\text{split}|\sum w_jC_j$ (identical parallel machines).

A number of scheduling problems with split jobs are polynomially solvable in the absence of setup times [11, 13], however when setup times are present these problems become NP-hard. Indeed, Schalekamp et al. [10] show that $P|\text{split}|\sum w_jC_j$ is NP-hard even with uniform setup times ($s_j = s$). They also design a 2.781-approximation algorithm for the uniform setup time version of $P|\text{split}|\sum C_j$ and an exact polynomial time al-