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Juan S. Borrero, Colin Gillen, Oleg A. Prokopyev

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A simple technique to improve linearized reformulations of fractional (hyperbolic) 0–1 programming problems

Juan S. Borrero[a] · **Colin Gillen**[a] · **Oleg A. Prokopyev**[a,b]

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Abstract We consider reformulations of fractional (hyperbolic) 0–1 programming problems as equivalent mixed-integer linear programs (MILP). The key idea of the proposed technique is to exploit binary representations of certain linear combinations of the 0–1 decision variables. Consequently, under some mild conditions, the number of product terms that need to be linearized can be greatly decreased. We perform numerical experiments comparing the proposed approach against the previous MILP reformulations used in the literature.

Keywords fractional 0–1 programming · hyperbolic 0–1 programming · linearization · binary representations · mixed integer linear programs

1 Introduction

In this paper we consider a general class of fractional 0–1 programs given by

$$\max_{x\in\mathcal{X}} \sum_{i=1}^{m} \frac{a_{i0}+\sum_{j=1}^{n} a_{ij}x_j}{b_{i0}+\sum_{j=1}^{n} b_{ij}x_j}, \tag{1}$$

where $\mathcal{X} := \{x \in \{0,1\}^n : Dx \leq d\}$ for given $D \in \mathbb{R}^{q\times n}$ and $d \in \mathbb{R}^q$. If $m = 1$ ($m > 1$), then the problem is also known as *single-ratio* (*multiple-ratio*). Note that if $\mathcal{X} = \{0,1\}^n$ ($\mathcal{X} \subset \{0,1\}^n$), then (1) is an *unconstrained* (*constrained*) fractional 0–1 program. Mathematical programming problems of the form (1) are often referred to as *hyperbolic 0–1 programs*, see, e.g., [14, 18].

Fractional 0–1 programs of the form (1) arise in applications such as cutting stock [10], set covering [6], scheduling [21] and information retrieval [13] problems. More recent applications include problems of a feature selection in biclustering [8, 25], wireless network design [5] and category pricing and assortment optimization in retail [17, 23].

If (1) is single-ratio, i.e., $m = 1$, and unconstrained, then it is solvable in polynomial time as long as $b_{10} + \sum_{j=1}^{n} b_{1j}x_j > 0$ for all $x \in \{0,1\}^n$, see [7, 14]. If $b_{10} + \sum_{j=1}^{n} b_{1j}x_j \neq 0$ for all $x \in \{0,1\}^n$, but can take on either positive or negative values, then the problem is NP-hard even for the single-ratio case [7, 14]. The unconstrained multiple-ratio case is NP-hard even if $m = 2$ and $b_{i0} + \sum_{j=1}^{n} b_{ij}x_j > 0$

[a]Department of Industrial Engineering, University of Pittsburgh, Pittsburgh, PA 15261, USA
[b]Laboratory of Algorithms and Technologies for Networks Analysis, National Research University Higher School of Economics, Nizhny Novgorod 603093, Russia

J.S. Borrero (Corresponding author): Tel.: 1-412-624-9830
E-mail: jsb81@pitt.edu

C. Gillen · O.A. Prokopyev
E-mail: {cpg12,droleg}@pitt.edu