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Exercising Control When Confronted by a (Brownian) Spider

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Abstract

We consider the Brownian "spider," a construct introduced in [4] and in [1]. In this note, the author proves the "spider" bounds by using the dynamic programming strategy of guessing the optimal reward function and subsequently establishing its optimality by proving its excessiveness.

Keywords: Dynamic programming, martingales, stochastic optimization, Brownian motion

2010 MSC: Primary: 90C39, Secondary: 90C15

In memory of my mentor, Professor Larry Shepp (1936-2013)

1. Introduction

In this note, we consider the Brownian "spider," a process also known as the "Walsh" Brownian motion, due to [4] and [1]. The Brownian spider is constructed as a set of $n \geq 1$ half-lines, or "ribs," meeting at a common point, O. A Brownian motion on a spider starting at zero may be constructed from a standard reflecting Brownian motion $(|W_t|, t \geq 0)$ by assigning an integer $i \in \{1, \ldots, n\}$ uniformly and independently to each excursion which is then transferred to an excursion on rib i (here, i should be interpreted as the index of the rib on which the next excursion occurs). It is helpful to think about the Brownian spider as a bivariate process; the first coordinate of the process is reflecting Brownian motion and the second coordinate of the process is the rib index. Formally, we define the Brownian spider process Z_t as

$$Z_t = (|W_t|, R_t)\,, t \geq 0 \tag{1}$$

where $|W_t|$ is reflected Brownian motion and R_t is the rib on which the process is located at time t. $|W_t|$ can be decomposed into excursions away from 0 with endpoints t_k s.t. $|W_{t_k}| = 0$. R_t is constant between t_k and t_{k+1} for all i, and $R_t = i$ means the excursion occurs on the rib i. We define the supremum of reflected Brownian motion on each rib as

$$S_i(t) = \sup_{\{t:\ R_t = i\}} |W_t|,\ t \geq 0,\ i = 1, ..., n.$$

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