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Stochastic capacity expansion with multiple sources of capacity

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ABSTRACT

In this paper, we consider the multi-period single resource stochastic capacity expansion problem with three sources of capacity: permanent, contract, and spot market. The problem is modeled as a multi-stage stochastic integer program. We show that the problem has the totally unimodular property and develop polynomial-time primal and dual algorithms to solve the problem.

1. Introduction

Capacity expansion is mainly concerned with the optimal time and amount of capacity acquisition, and the optimal capacity allocation. Capacity expansion models can be found in a wide range of applications as a strategic level decision, which usually involves significant capital investments, as well as uncertainties in the future forecasts. Traditionally, the models developed for capacity expansion assume that there is only one type of capacity available for acquisition. This capacity is purchased and permanently owned by the decision maker and it is called permanent capacity. However, in the real world, there could be other sources of capacity that the decision maker can use. In this paper, we introduce two commonly used sources of capacity available to decision makers besides permanent capacity: spot market capacity, which refers to the capacity that can only be purchased and used in the current period, and contract capacity, which refers to the capacity that is available in the current period—if a contract has been signed for it in previous periods. The quantity of contract capacity is assumed fixed in the contract. In this paper, we consider a multi-period single resource stochastic capacity expansion problem where all three sources of capacity exist simultaneously, and we model the problem using the multi-stage stochastic programming approach. It is noteworthy that our model can capture the case where the decision maker can sign contracts for an arbitrary number of periods.

The capacity expansion problem and its extensions have been extensively studied [8,12]. For single resource problems, Saniee [11] modeled a deterministic multi-period problem as a time-dependent knapsack problem. Laguna [4] extended this work to the case of uncertain demand. Riis and Andersen [10] considered the stochastic version of the single resource problem and proposed a two-stage formulation. Huang and Ahmed [3] studied a multi-stage stochastic version of capacity expansion for a single resource, in which stochastic demand and cost are represented by a scenario tree. Ahmed et al. [1] and Huang and Ahmed [3] also considered the lot-sizing problem as a substructure of general capacity expansion problems. However, all of these works only consider permanent capacity.

There is a fairly small literature for capacity expansion in the presence of spot market and contract. Atamtürk and Hochbaum [2] considered various versions of deterministic multi-period capacity expansion problem when subcontracting is available. Oren et al. [9] considered electric power capacity expansion with spot market in a game-theoretical framework. Other works can be found in [5–7] with applications in cellular manufacturing and flexible manufacturing systems. All of these works assume deterministic demand and cost.

In the following, we use a scenario tree with T periods to represent the realization of stochastic parameters (demand, cost, etc.), as shown in Fig. 1. For each node n, $a(n)$ is the immediate ancestor node, $\mathcal{C}(n)$ is the set of immediate descendants, and t_n is the period of node n. Let $\mathcal{T}$ be the whole scenario tree and N be the total number of nodes in the scenario tree. $\mathcal{T}(n)$ denotes the subtree with root node n and $\bar{\mathcal{T}}(n) = \mathcal{T}(n) \setminus \{n\}$. $\mathcal{P}(n)$ denotes the

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unique path from node n to the root node of the scenario tree, and $\bar{\mathcal{P}}(n) = \mathcal{P}(n) \setminus \{n\}$. We also use the following notations in the model:

Parameters:

c_n^p Unit cost of permanent capacity for node n,
c_n^c Unit cost of contract capacity for node n,
c_n^s Unit cost of spot market capacity for node n,
p_n Probability of node n,
d_n Demand of node n.

Decision variables:

x_n Permanent capacity acquisition in node n,
y_n Contract capacity acquisition in node n,
z_n Spot market capacity acquisition in node n.

We assume that we do not have any previously purchased or contracted capacity. We also assume that all costs and demands are positive and all costs are discounted to their present value. Moreover, we assume that c_n^p and c_n^c are very large values for leaf nodes. With these assumptions, the stochastic single resource capacity expansion problem can be formulated as:

$$\begin{aligned} \text{Min} \quad & \sum_{n\in\mathcal{T}} p_n \left(c_n^p x_n + c_n^c y_n + c_n^s z_n\right) \\ \text{s.t.} \quad & \sum_{m\in\bar{\mathcal{P}}(n)} x_m + y_{a(n)} + z_n \geq d_n \quad \forall\, n \in \mathcal{T}, \\ & x_n, y_n, z_n \in \mathbb{Z}^+ \quad \forall\, n \in \mathcal{T}, \end{aligned} \tag{1}$$

where the objective minimizes the expected total cost of all three types of capacity acquisition over the scenario tree, and the first set of constraints guarantee that for each node in the scenario tree, the total capacity available at node n will satisfy the demand. Note that we purchase the permanent capacity or sign the contract capacity at the beginning of the period, before the realization of the random demand in the same period. Thus, in model (1), we make permanent or contract capacity available in the next period of when it is purchased or signed. In other words, x_n will appear in $\bar{\mathcal{T}}(n)$ and y_n will appear in $\mathcal{C}(n)$. We emphasize that our model can deal with contracts with an arbitrary number of periods. For simplicity of exposition, we only use the one-period contract in model (1). Also, it can be verified that a lot-sizing problem with spot market and contract production availability can be transformed to an equivalent model (1) in polynomial time. Model (1) has the following property:

Theorem 1. *If the demands are integer-valued, the LP relaxation of model (1) will yield integral optimal solutions.*

Proof. Suppose that A is the left hand side matrix of model (1). Note that A is a 0-1 matrix and each row in A corresponds to a node in the scenario tree. Consider the z_n columns, there is a single 1 in each column and all other entries are zeroes. Consider the y_n columns, the entry of row i and column j is 1 only if $i \in \mathcal{C}(j)$. Consider the x_n columns, the entry of row i and column j is 1 only if $i \in \bar{\mathcal{T}}(j)$. We can reorder the constraints according to the following procedure: we start from the root node and after each insertion of a node n, we immediately insert all nodes in $\mathcal{C}(n)$. If there is more than one node in $\mathcal{C}(n)$, we will re-start the process from the first inserted node. When there are no more nodes to insert, we continue the procedure from the last non-inserted node based on a depth-first search method. This new ordering guarantees that in each column, the 1s appear consecutively. Therefore, A is an interval matrix which is totally unimodular [13]. □

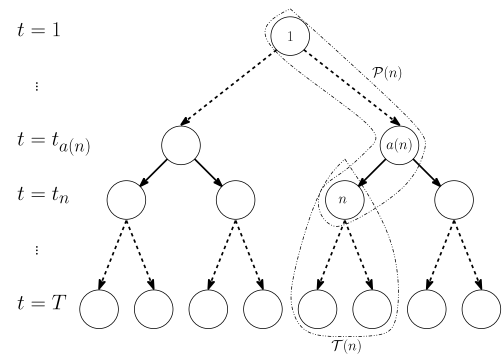

Fig. 1. The scenario tree $\mathcal{T}$.

Based on Theorem 1, we can rewrite model (1) as follows:

$$\begin{aligned} \text{Min} \quad & \sum_{n\in\mathcal{T}} \left(c_n^p x_n + c_n^c y_n + c_n^s z_n\right) \\ \text{s.t.} \quad & \sum_{m\in\bar{\mathcal{P}}(n)} x_m + y_{a(n)} + z_n \geq d_n \quad \forall\, n \in \mathcal{T}, \\ & x_n, y_n, z_n \in \mathbb{R}^+ \quad \forall\, n \in \mathcal{T}. \end{aligned} \tag{2}$$

Note that for simplicity of exposition, we have removed the p_ns in the objective. In the following, all our algorithms are designed for model (2).

2. Algorithms

2.1. Primal algorithm

In this subsection, we propose a primal algorithm for model (2). The primal algorithm will check if it is more economical to shift up capacity to an ancestor node, as either permanent capacity or contract capacity, by comparing the costs of permanent capacity or contract capacity in an ancestor node with the total cost of permanent, contract, and spot market capacities in descendant nodes.

For this algorithm, we assume that the nodes in the scenario tree $\mathcal{T}$ are indexed in increasing order of their time periods (called the *primal indexing system*). The algorithm starts with an initial feasible solution, where $x_n^* = y_n^* = 0$ and $z_n^* = d_n$ for all $n \in \mathcal{T}$. We also need the following definitions:

$$\mathcal{A}^1(n) = \left\{m \in \bar{\mathcal{T}}(n) : x_m > 0, \text{ and } x_k = 0, \forall k \in \bar{\mathcal{P}}(m) \setminus \mathcal{P}(n)\right\}$$

$$\mathcal{A}^2(n) = \left\{m \in \bar{\mathcal{T}}(n) : y_m > 0, \text{ and } x_k = 0, \forall k \in \mathcal{P}(m) \setminus \mathcal{P}(n)\right\}$$

$$\mathcal{A}^3(n) = \Big\{m \in \bar{\mathcal{T}}(n) : z_m > 0, \text{ and } x_k = y_k = 0, \\ \forall\, k \in \bar{\mathcal{P}}(m) \setminus \mathcal{P}(n)\Big\}$$

$$\mathcal{A}(n) = \mathcal{A}^1(n) \cup \mathcal{A}^2(n) \cup \mathcal{A}^3(n)$$

$$\Delta_n^1 = \text{Min}\left\{\underset{m\in\mathcal{A}^1(n)}{\text{Min}}\ x_m,\ \underset{m\in\mathcal{A}^2(n)}{\text{Min}}\ y_m,\ \underset{m\in\mathcal{A}^3(n)}{\text{Min}}\ z_m\right\}$$

$$\Delta_n^2 = \underset{\substack{m\in\mathcal{C}(n)\\ z_m>0}}{\text{Min}}\ z_m$$

$$B_n^1 = \sum_{m\in\mathcal{A}^1(n)} c_m^p + \sum_{m\in\mathcal{A}^2(n)} c_m^c + \sum_{m\in\mathcal{A}^3(n)} c_m^s - c_n^p$$