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Economic lot sizing: The capacity reservation model

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ABSTRACT

Capacity reservation contracts allow a consumer to purchase up to a certain capacity at a unit price lower than that of the spot market, while the consumer's excess orders are realized at the spot price. In this paper, we consider a lot sizing problem where the consumer places orders following a capacity reservation contract. In particular, we study the general problem and the polynomial time solvable special cases of the problem and propose corresponding algorithms for them.

1. Introduction

Consider a scenario where a retailer (she) keeps a long-term contract with a manufacturer (he). In return for the retailer's regular purchases, the manufacturer offers the retailer products up to a given capacity at a unit price lower than the spot price. When the retailer's desired procurement quantity exceeds the reservation capacity, she has to realize the excess quantity at the spot price from the same supplier. Known as *capacity reservation contracts*, such contracts are extensively used for purchasing chemicals, commodity metals, semiconductors and electric power [18]. The capacity reservation model can also be applied to production models where extra cost is incurred due to overtime production or the outsourcing part of production.

Formally, we consider the capacity reservation contracts with two parameters, c, the unit purchasing price specified by the capacity reservation contract, and Q, the given capacity. Suppose that the fixed ordering cost is K, the spot price is s, and the retailer's procurement quantity is q. Then, to realize all her demand, the retailer's purchasing cost is

$$Y(q) = \begin{cases} 0 & q = 0 \\ K + cq & 0 < q \le Q \\ K + cQ + s(q - Q) & Q < q \end{cases} \quad (1)$$

or we could equivalently write $Y(q) = K\mathbf{1}_{\{q>0\}} + cq + (s-c)(q-Q)^+$, where $(\cdot)^+ = \max\{\cdot, 0\}$.

Capacity reservation contracts are frequently used in procurement and transportation. For example, Jin and Wu [17] and Erkoc and Wu [10] consider the capacity reservation contracts for the high-tech industry. van Nordena and van de Veldeb [22] first study the dynamic lot sizing problem with a transportation capacity reservation contract. Inderfurth and Kelle [16] consider the combined use of capacity reservation contracts and the spot market in the newsvendor model.

Similar contracts are also studied by several researchers. Henig et al. [14] study a periodic-review inventory-control model. In their model, when the order quantity is below a given volume, the ordering cost is zero, otherwise the cost is linear in the exceeding quantity. Chao and Zipkin [4] consider a similar problem, where a fixed cost is incurred if the order quantity is above the volume. Caliskan-Demirag et al. [3] study a periodic-review inventory problem where the fixed cost depends on the order quantity.

Our work, on the other hand, is an extension of the economic lot sizing problem, with deterministic and time varying demand, capacity and cost parameters. The goal is to find a plan that minimizes the total inventory and procurement cost. Wagner and Whitin [24] first develop an $O(T^2)$ dynamic programming algorithm for the general lot sizing problem, also known as the WW problem. Later research works focus on studying algorithm complexity for different models; see [20] for example. The $O(T^2)$ dynamic programming algorithm was improved independently by Aggarwal and Park [1], Federgruen and Tzur [11] and Wagelmans et al. [23] who developed an $O(T \log T)$ algorithm for the general problem. The capacitated lot sizing problem (CLSP) can be viewed as a generalization of the WW problem. Known to be $\mathcal{NP}$-hard [2,13], many heuristics are designed for the CLSP problem [8,15]. Readers might refer to [9] for a survey of lot sizing problems.

The capacity reservation model discussed in this paper could be conveniently viewed as an extension of the CLSP problem, since an algorithm for the lot sizing problem with capacity reservation (LS-CR) could always be applied to a capacitated lot sizing problem by setting the spot price to infinity at every time slot. The LS-CR

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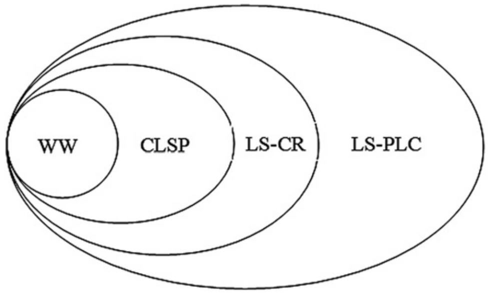

Fig. 1. The relationship between different problems.

Table 1
Complexity of different models.

	$NI/G/NI/ND$	$G/G/G/C$
Traditional model	$O(T^2)$[a]	$O(T^3)$[b]
Our model	$O(T^3)$	$O(T^4)$

[a] See [6] for details.
[b] See [22] for details.

problem, on the other hand, is a special case of the lot sizing problem with piecewise linear production costs (LS-PLC). Chen et al. [5] present an efficient dynamic programming algorithm for the general LS-PLC problem with computational results, and Shaw and Wagelmans [19] propose a pseudo-polynomial time algorithm. We characterize the relationship between the different lot sizing problems in Fig. 1.

Although the capacitated problems are quite difficult to solve in general, many special cases have polynomial time algorithms. Bitran and Yanasse [2] design a classification scheme for the capacitated lot sizing problem, and introduce the four field notation $\alpha/\beta/\gamma/\delta$, where α, β, γ and δ represent the setup cost (fixed ordering cost in our model), unit holding cost, unit production cost (unit purchasing cost from long-term contract and spot market price in our model), and capacity type (reservation quantity in our model), respectively. Each of the parameters might have an arbitrary pattern (general, G), be constant (C), nondecreasing (ND), nonincreasing (NI) or zero (Z). Different from the scheme, we allow unlimited purchase from the spot market (at a potentially higher price) when the reservation quantity is exhausted, and thus a feasible solution always exists. It should be noted that when $\gamma = C$ (resp., NI, ND), both the unit purchasing prices specified in the capacity reservation contract and the spot prices are stationary (resp., nonincreasing, nondecreasing) over all time periods.

In this paper, we focus on the following two models: $NI/G/NI/ND$ and $G/G/G/C$, both of which are known to have polynomial time algorithms in the classic model but not known in the capacity reservation model. In this work, we show that both models have polynomial time algorithms in the capacity reservation model, and we present our results in Table 1.

2. Problem formulation and computational complexity

Consider the problem with T horizons, and the capacity reservation contract specifies unit purchasing cost c_i and capacity q_i for period $i \in \{1, \ldots, T\}$. The inventory holding cost for period i is h_i, while backorders are not allowed. The demand in period i is d_i, which is known prior to the making of the purchase decision. In the general model, the fixed ordering cost for period i is K_i. The spot price in time i is s_i, where $s_i > c_i$, implying that the spot price is always higher than price within the given capacity. All costs are nonnegative and the capacity is strictly positive.

The decision variables q_i are the purchasing quantities in periods i, $i = 1, \ldots, T$, and the objective is to minimize the purchasing costs and inventory costs,

$$\min \sum_{i=1}^{T} K_i 1_{\{q_i>0\}} + c_i q_i + (s_i - c_i)(q_i - Q_i)^+ + h_i I_i$$

where I_i represents the inventory level at the end of period i. The program is subject to the following constraints:

- Balance of the inventory:

  $I_{i-1} + q_i - d_i = I_i$.
- Nonnegativity:

  $q_i \geq 0, \qquad I_i \geq 0$.
- And the initial inventory level is zero:

  $I_0 = 0$.

The dynamic lot sizing problem without fixed ordering cost is the case $K_i = 0$ for every i.

The general lot sizing problem with capacity reservation is $\mathcal{NP}$-hard due to the $\mathcal{NP}$-hardness of its special case, i.e., the capacitated lot sizing problem. In fact, the problem remains $\mathcal{NP}$-hard even if we put stronger restrictions on the problem, including the classes $C/Z/NI/NI$, $C/Z/ND/ND$, $ND/Z/Z/ND$, $NI/Z/Z/NI$, $C/G/Z/NI$ and $C/C/ND/NI$; see [2] for proofs.

A straightforward dynamic programming algorithm could solve the general problem in time $O(T^3\overline{d}^2)$, where $\overline{d}$ denotes the average demand over all periods.

More efficient algorithms could be explored for the general problem. Shaw and Wagelmans [19] show that the capacitated lot sizing problem with piecewise linear production cost functions can be solved in $O(T^2\overline{q}\overline{d})$ time, where $\overline{q}$ is the average number of pieces needed to represent the linear cost function.

As discussed before, it would be convenient to view the lot sizing problem in the capacity reservation model as a special case of the piecewise cost function model; therefore we immediately come up with the following result: there exists an $O(T^2\overline{d})$ algorithm for the lot sizing problem in the capacity reservation model. Since the general problem is $\mathcal{NP}$-hard, this pseudo-polynomial time algorithm is quite useful.

When there is no fixed ordering cost, the lot sizing problem is solvable in time $O(T \log T)$ by simply keeping a sorted list of available purchasing alternatives.

3. The *NI/G/NI/ND* class reservation problem

Although the lot sizing problem in the capacity reservation model is $\mathcal{NP}$-hard in general, it is still possible to solve some special cases of the problem quite efficiently. Bitran and Yanasse [2] first design an $O(T^4)$ algorithm for the $NI/G/NI/ND$ capacitated lot sizing problem, and Chung and Lin [6] improve their result to $O(T^2)$. Motivated by their work, we study the $NI/G/NI/ND$ class problem in the capacity reservation model.

For the $NI/G/NI/ND$ problem, over the time, the setup costs K_i are nonincreasing, the unit holding costs have arbitrary pattern, the unit purchasing cost c_i and s_i are nonincreasing and the capacities Q_i are nondecreasing. The traditional approaches (cf. [6]) for the capacitated lot sizing problem use the idea of a subplan, and so do we.

Definition 1. A subplan S_{uv} is the part of plan covering the demand from $u + 1$ to v such that $I_u = 0, I_v = 0$ and $I_t > 0$ for any $u < t < v$.

To develop our algorithm for the $NI/G/NI/ND$ class, observe that

$$f(t) = \min_{1 \leq u < t} \{f(u) + C(S_{ut})\} \qquad (2)$$

where $C(S_{ut})$ is used to denote the minimum cost of the subplan S_{ut}, while $f(t)$ represents the optimal objective function value for