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Asymptotically optimal schedules for single-server flow shop problems with setup costs and times

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Abstract

We consider the processing of M jobs in a flow shop with N stations in which only a single server is in charge of all stations. We demonstrate that for the objective of minimizing the total setup and holding cost, a class of easily implementable schedules is asymptotically optimal.

Keywords: Flexible machine scheduling; Setup times; Asymptotic optimality; Flow shop

1. Introduction

This paper focuses on a basic scheduling problem which arises in production environments with cross-trained workers or flexible machines. Consider a serial production line consisting of N stations in which only a single worker (server) is in charge of all stations (see Fig. 1). Suppose M jobs must be processed in this line, and they will be shipped out of the system as soon as all M jobs are completed. All jobs require the operations in all stations in the sequence $1, 2, \ldots, N$, and

- p_i denotes the job processing time at station (stage) i.
- h_i denotes the unit holding cost rate at station i, where h_{N+1} is the unit holding cost rate for completed jobs awaiting shipment. We assume that h_i is non-decreasing in i, due to the value added to a job each time an operation is completed on the job, $i = 1, 2, \ldots, N+1$.
- K_i and U_i denote the setup cost and setup time incurred, respectively, whenever the worker switches from station $j \neq i$ to station i.

Without loss of generality, we assume that a setup time of U_1 and a setup cost K_1 are also incurred when the worker begins to work on these M jobs.

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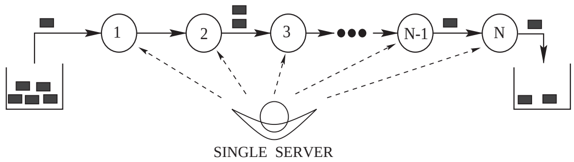

Fig. 1. Single-server flow shop scheduling problem.

Our objective is to find an optimal worker schedule that minimizes the *total holding and setup cost of processing all M jobs*. In other words, any time that the worker finishes a job at a station, we want to determine whether the worker's best action is to (i) switch to the next station and perform the next operation on the same job, (ii) process another job (if there is any) at the same station, or (iii) switch to another station and process another job.

Our problem is in fact a *flow shop scheduling problem with only one worker*. In standard flow shop problems there is a dedicated worker for each station in the line, so different jobs can be simultaneously processed in different stations. However, in our problem there is only one worker assigned to all stations, and thus, only one job can be processed at any time. Our problem can also be viewed as a *single flexible machine scheduling* problem as follows: M jobs require N different operations in the sequence of $1, 2, \ldots, N$ on a single flexible machine. The jobs must be shipped out together as soon as all of them are completed. The machine is capable of performing all M operations, but it requires a time U_i for setup, and incurs a setup cost K_i whenever it switches from another operation to operation i. Operation i takes p_i units of time, and holding cost rate h_i is charged for each job awaiting or undergoing operation i.

Iravani et al. [6] considered a similar scheduling problem with setup costs but zero setup times. They focused on a class of schedules, which they called "chain-structure" schedules, and using extensive *numerical experiments* they concluded that the cost of the optimal chain-structure schedule is close to the cost of the global optimal schedule. This paper looks at a more general case with setup costs and *non-zero setup times*. Furthermore, the paper combines a technique from scheduling reversibility and a 2D Gantt Chart to present an *analytical proof* for the optimality of chain-structure schedules as $M \to \infty$. We also show that every algorithm in a natural class of algorithms is a 2-approximation algorithm for the single-server flow shop problem with setup costs and setup times.

2. Literature review

By now a huge literature exists on machine and flow shop scheduling problems. We refer readers to Pinedo [10] for a general review of the single machine and flow shop scheduling literature. Flow shop scheduling with a single worker and non-zero setup times was considered in [6]. (See [6] for a review of studies on single-server flow shop scheduling in stochastic environments, i.e., tandem queues attended by moving servers.) Brucker and Schlie [3] and Brucker et al. [2] extended the classical scheduling models (from parallel machines to job shop environments) by assuming that machines are general purpose, i.e., jobs or operations can be processed by any machine in a prespecified subset of machines. This introduces routing complexity in the scheduling problem and leads to a substantially more complex model. The flexible machine version of our model is different in that only a single flexible machine is used for all operations. Furthermore, all jobs are shipped together at the end of all the operations.

Asymptotic optimality results have been discovered for a variety of other machine scheduling problems (e.g., [8,11,12] to name a few). In fact, Bertsimas et al. [1] obtained asymptotically optimal policies for a general job shop problem (including the re-entrant job shop, which includes our problem as a special case) under the minimum holding cost objective. Unfortunately, these results were obtained via an intricate analysis and do not readily generalize to the case with setups. While the ideas described here can be combined with their techniques to establish our main results, we show that such an intricate analysis is not necessary for the problem we consider. In fact, we show how a simple 2D Gantt chart can be used to derive the same result.

The key observation we exploit is the following: if there are M jobs to be processed, and the jobs must be released as a batch, the holding cost of any reasonable schedule is $cM^2 + o(M^2)$. In our algorithm, we make sure that (i) setup costs and the additional holding costs due to setup times are $o(M^2)$, and (ii) the