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Item response theory for longitudinal data: population parameter estimation

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Abstract

In this work we propose IRT models to estimate ability distribution parameters of a population of individuals submitted to different tests along the time, having or not common items. The item parameters are considered known and several covariance structures are proposed to accommodate the possible dependence among the abilities of the same individual, measured at different instants. Maximum likelihood equations and some simulation results are presented.

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1. Introduction

In most IRT applications we are interested more in estimating ability distribution parameters than abilities of the individuals. In general, the proposed estimates are functions of the estimated abilities of the individuals. Andersen and Madsen [1] showed how it could be done directly, without having to estimate the abilities of the individuals, in situations when the item parameters are known. Some subsequent works (see Sanathanan and Blumenthal

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[17]; de Leeuw and Verhelst [14]; and Lindsay et al., [15]) extended their results to situations when it is necessary to estimate both item and ability distribution parameters. These extensions were appropriate for the one parameter logistic model with individuals belonging to a single population. Bock and Zimowski [3] introduced the multiple groups models that extended these results for the 1, 2 and 3 parameter logistic models with individuals belonging to more than one, but independent, populations. The reader can see Lord [16], Hambleton et al. [11] and Andrade et al. [2], among others for details on the fundamentals and applications of IRT.

A different situation occurs when the same individual is evaluated along time. This is the case of a study carried out by the Ministry of Education of Brasil, that was planned to follow students from 4th to 8th grade. At the end of each one of the five years, the students were submitted to a test. They were also evaluated at the beginning of the first year. It is natural to think that there is a dependence structure among the abilities along time. In this work we propose IRT models to evaluate one group of individuals at T instants of evaluation. For instance, in T consecutive years. At each instant, the individuals are submitted to a different test with n_t items each, having or not common items. The total number of items n will be less than or equal to $n_c = \sum_{t=1}^{T} n_t$. It will be assumed that all the items parameters are known, for instance when the tests are built with items from an item bank with items calibrated on the same metric. In Section 2, we introduce the models and in Section 3, the likelihood equations for each one of the 6 covariance structures proposed to accommodate for the dependence between abilities of the individuals along time. Results from a simulated study are presented in Section 4. Final comments and suggestions are presented in Section 5.

2. The model

Let

$$P_{jit} = P(U_{jit} = 1|\theta_{jt}, \zeta_i)$$

be a twice-differentiable item response function that describes the conditional probability of a correct response to item $i,\ i = 1, 2, \ldots, n$, of individual $j,\ j = 1, 2, \ldots, N$, in test $t,\ t = 1, 2, \ldots, T$, where U_{jit} represents the (binary) response, θ_{jt} the ability (latent trait) and ζ_i the known vector of the item parameters. Examples of such a function are the logistic 1, 2 and 3 parameters models (see Hambleton et al. [11] for details). Assuming the conditional independence of the responses to the items in test t, given θ_t, we have that

$$P(\boldsymbol{U}_{j.t}|\theta_t, \boldsymbol{\zeta}) = \prod_{i \in \boldsymbol{I}_t} P(U_{jit}|\theta_t, \zeta_i),$$

where $\boldsymbol{I}_t$ is the set of the indexes of those items presented in test t, $\boldsymbol{U}_{j.t} = (U_{j1t}, U_{j2t}, \ldots, U_{jn_t t})'$ is the $(n_t \times 1)$ vector of responses of individual j in test t and $\boldsymbol{\zeta} = (\zeta_1', \zeta_2', \ldots, \zeta_n')'$ the known vector of the items parameters. Note that, for convenience and without loss of generality we dropped index j from the ability parameter because we are interested in the distribution of the ability at each instant t and not in any particular θ_{jt}. Furthermore,