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Journal of Multivariate Analysis 93 (2005) 434–445
Journal of Multivariate Analysis
http://www.elsevier.com/locate/jmva

Archimedean copulae and positive dependence

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Received 25 June 2003

Available online 19 May 2004

Abstract

In this paper, we consider different issues related to Archimedean copulae and positive dependence. In the first part, we characterize Archimedean copulae that possess positive dependence properties such as multivariate total positivity of order 2 (MTP_2) and conditionally increasingness in sequence. In the second part, we investigate conditions for exchangeable binary sequences to admit an Archimedean copula, and we show that they depend on the extendibility of the sequence, and therefore on its positive-dependence properties.
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AMS 2000 subject classifications: primary 60E15; secondary 60G09; 62H10; 62H20

Keywords: Conditionally increasing; MTP_2; Positive lower orthant dependent; Exchangeability; Binary sequences

1. Introduction

Archimedean copulae have recently received increasing attention for their computational tractability and their flexibility to model dependencies between exchangeable random variables. They have been introduced by Kimberling [22]. Some basic contributions have been given by Genest and MacKay [15], Marshall and

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0047-259X/$ - see front matter © 2004 Elsevier Inc. All rights reserved.
doi:10.1016/j.jmva.2004.04.003

Olkin [25], Ballerini [3], and Oakes [29]. More specific results about Archimedean copulae have been given, among others, by Genest and Rivest [17], who studied statistical inference, Bagdonavičius et al. [2], who provided characterizations, and Juri and Wüthrich [20] proved convergence theorems for tail events. Cuculescu and Theodorescu [8] investigate unimodality of bivariate Archimedean copulae. In recent years these copulae have been used more and more as a tool for modelling dependence in many diverse areas. Frees and Valdez [13] demonstrated their usefulness for problems in actuarial sciences. In the literature on financial mathematics they have been considered in [11,18].

In this paper, we will characterize several well-known notions of positive dependence in the case of Archimedean copulae. For the case of positive orthant dependence such a result can be found in [19]. A sufficient condition for supermodular ordering of Archimedean copulae (and thus also for supermodular positive dependence) is given in [34], see also [10,16]. Some results for the bivariate case can also be found in [6,7] and in [1]. Bassan and Spizzichinao [5] study dependence and aging concepts for bivariate distributions in terms of an object that they call Archimedean semi-copula.

In this paper, we deal with the multivariate case. In particular, we derive necessary and sufficient conditions for the generator of an Archimedean copula to yield a random vector which is MTP_2 or conditionally increasing in sequence. For these two important notions of positive dependence the reader is referred for instance to [4,21].

Then, we consider the particular case of binary random variables. We show that any infinitely extendible exchangeable vector of binary random variables admits a representation by an Archimedean copula. We also prove that this is not true for finitely extendible exchangeable vectors. Extendibility is a positive dependence issue, since it is well-known that infinitely extendible exchangeable sequences cannot exhibit negative dependence (the covariance of any pair of them is non-negative, see [12]).

Due to de Finetti's representation theorem, infinite exchangeable binary sequences are extremely relevant, for instance in Bayesian statistics. It is therefore important to know that their distribution can always be represented by an Archimedean copula.

The rest of the paper is organized as follows. In Section 2, we first give the basic definitions on copulae and notions of positive dependence, and then state the main results characterizing these concepts of dependence for Archimedean copulae. In Section 3, we investigate the relation between exchangeable binary sequences and Archimedean copulae.

2. Copulae and positive dependence

The notion of copula has been introduced by Sklar [33], and studied, among others, by Kimeldorf and Sampson [23], under the name of uniform representation, and by Deheuvels [9], under the name of dependence function. The copula is one of the most useful tools for handling multivariate distributions with dependent components. Formally, given a distribution function F with marginals $F_1, \dots, F_d$,

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