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Improving on the mle of a bounded location parameter for spherical distributions

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Abstract

For the problem of estimating under squared error loss the location parameter of a p-variate spherically symmetric distribution where the location parameter lies in a ball of radius m, a general sufficient condition for an estimator to dominate the maximum likelihood estimator is obtained. Dominance results are then made explicit for the case of a multivariate student distribution with d degrees of freedom and, in particular, we show that the Bayes estimator with respect to a uniform prior on the boundary of the parameter space dominates the maximum likelihood estimator whenever $m \leqslant \sqrt{p}$ and $d \geqslant p$. The sufficient condition $m \leqslant \sqrt{p}$ matches the one obtained by Marchand and Perron (Ann. Statist. 29 (2001) 1078) in the normal case with identity covariance matrix. Furthermore, we derive an explicit class of estimators which, for $m < \sqrt{p}$, dominate the maximum likelihood estimator simultaneously for the normal distribution with identity covariance matrix and for all multivariate student distributions with d degrees of freedom, $d \geqslant p$. Finally, we obtain estimators which dominate the maximum likelihood estimator simultaneously for all distributions in the subclass of scale mixtures of normals for which the scaling random variable is bounded below by some positive constant with probability one.

AMS 2000 subject classifications: 62F10; 62F15; 62F30

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1. Introduction

Consider the problem of estimating under squared error loss the location parameter θ of a spherically symmetric distribution, based on the observation X and with the constrained parameter space $\Theta(m) = \{\theta \in \mathbb{R}^p\colon ||\theta|| \leqslant m\}$ for some m fixed, $m > 0$. In the normal case with identity covariance matrix, Marchand and Perron [4] showed that, among classes of dominating estimators, the Bayes estimator δ_{BU} with respect to the boundary uniform prior on $\partial\Theta(m)$ dominates the maximum likelihood estimator δ_{mle} whenever $m \leqslant \sqrt{p}$. An interesting question is to investigate whether similar dominance results hold for other spherically symmetric distributions. This is indeed the objective of our research, and our results are focussed on: (i) the multivariate student distribution which represents perhaps one of the most important alternatives to the normal model, and (ii) on scale mixtures of normals for which the scaling random variable is bounded below by some positive constant with probability one.

The starting point of our inquiry is a sufficient condition (Theorem 1) for an estimator to dominate δ_{mle}, which was implicitly given by Marchand and Perron [4, Theorem 3], and which is applicable in general to spherically symmetric distributions. In Section 3.2, we provide explicit dominance conditions applicable to multivariate student distributions. We also study how these conditions apply to δ_{BU}, and establish (Example 1) that the condition $m \leqslant \sqrt{p}$ is, whenever $d \geqslant p$, once again sufficient for δ_{BU} to dominate δ_{mle}. The common sufficient condition is interesting and somewhat surprising, in view of its simplicity, and the fact that both the functional form of the estimator δ_{BU} and the distribution under which the risks are evaluated vary with d.

We also can view the sufficient condition for dominance of Theorem 1 as a sufficient condition for simultaneous dominance (Theorem 2), meaning a condition under which a single estimator δ_0 dominates δ_{mle} simultaneously for a subclass of spherical distributions. Of course, it is the hope that such a simultaneous condition of dominance can be made explicit for important subclasses of spherical distributions, possibly including the normal case. Simultaneous dominance is an appealing property in view of the intrinsic motivation of assessing or searching for procedures that retain good or optimal properties over a range of probability models. Although there seems to be a relative paucity of results in this direction, this is not a new theme; for instance, a fair amount of work on estimating a multivariate mean (without constraints) has dealt with procedures that are robust, in the sense that they perform well not only for the normal model, but also for a range of spherical or elliptical models. As a first example, Cellier and Fourdrinier [1], gave a class of estimators that dominate the unbiased estimator, for $p \geqslant 3$, simultaneously for all spherically symmetric distributions subject to (weak) risk finiteness conditions. A second example is given by the work of Srivastava and Bilodeau [7] who demonstrate robust dominance properties of the Stein estimator (in dominating the unbiased estimator) for elliptical distributions which are scale mixtures of normals.