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On the asymptotic properties of multivariate sample autocovariances

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Abstract

We show that if a process can be obtained by filtering an autoregressive process, then the asymptotic distribution of sample autocovariances of the former is the same as the asymptotic distribution of linear combinations of sample autocovariances of the latter. This result is used to show that for small lags the sample autocovariances of the filtered process have the same asymptotic distribution as estimators utilizing more information (observations on the associated autoregression process and knowledge of the parameters of the filter). In particular, for a Gaussian ARMA process the first few sample autocovariances are jointly asymptotically efficient.

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1. Introduction

The sample autocovariances are widely used in time series analysis, they are easy to compute and it is of interest to know how efficient they are.

Under quite general conditions (see Section 5), any fixed vector of sample autocovariances is asymptotically normal and the elements of the asymptotic covariance matrix are given by the Bartlett's formulas plus, in the non-Gaussian case, terms involving fourth-order cumulants (for details see [1,5, Chapter 8]). In the

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multivariate case the infinite sums in Bartlett's formulas can be replaced by the autocovariances corresponding to the tensor square of the spectral density matrix of the process (see [3]). For univariate processes the latter is simply the square of the spectral density (see [2,4,10]).

For Gaussian parametric models the Cramer–Rao bound for estimators of the autocovariances can be obtained by an appropriate transformation of the Cramer–Rao bound for the parameters. The latter can be found in [10] for the univariate case, and [7] for multivariate autoregressive moving average (ARMA) models. A direct comparison of the asymptotic covariance matrix of the sample autocovariances with the Cramer–Rao bound has been used by Porat [9] to show that the sample autocovariances for univariate ARMA(p, q) models are jointly asymptotically efficient up to lag $p - q$, provided that $p \geqslant q$, and are inefficient otherwise. Walker [11] derived this result by considering an approximation to the likelihood of the sample autocovariances. Porat [10] extended his earlier result (see [9]) to Gaussian processes other than ARMA by giving a necessary and sufficient condition for asymptotic efficiency of sample autocovariances in terms of the spectral density and its derivatives.

The method of Porat [10] was extended to multivariate processes by Kakizawa [6]. He did not consider joint efficiency for several lags although his method should be applicable to that problem too. Kakizawa was able to conclude that in the case of pure autoregressions of order p the autocovariances up to lag p are asymptotically efficient while for the pure moving average none of the autocovariances is asymptotically efficient. The mixed ARMA case remained open.

The efficiency of the sample autocovariances up to lag p for (univariate or multivariate) autoregressive models of order p, can be derived also indirectly from the efficiency of the least-squares and Yule–Walker estimators of the parameters of such models.

Comparisons with Cramer–Rao bound restrict the above methods to Gaussian processes. Our approach is to consider a class of models where the observed process is obtained by a finite linear transformation of a pure autoregressive process of order, say, p. In the univariate case this class of models is equivalent to the standard form of the ARMA model. In the multivariate case it provides one possible parameterization of the multivariate ARMA model.

We show that the sample autocovariances of the observed process have the same asymptotic distribution as (for brevity we say: are as good as) a linear combination of the sample autocovariances of the underlying autoregressive process (Theorem 1 and Corollary 1). Hence, when this linear combination involves only lags between 0 and p, the corresponding sample autocovariances of the observed process are as good as estimators that would be available if both, the underlying autoregression was observed, and the parameters of the linear combination were known. This result seems rather strong since the autocovariance and least-squares estimators for autoregressions are asymptotically equivalent. In the Gaussian case asymptotic efficiency of the sample autocovariances of the observed process follows from the fact that the first $p + 1$ autocovariances of the underlying autoregressive process are asymptotically efficient (Theorems 2 and 3). The results are based on