Available online at www.sciencedirect.com

SCIENCE DIRECT®

Journal of Multivariate Analysis 95 (2005) 345–369

Journal of Multivariate Analysis

www.elsevier.com/locate/jmva

A multivariate empirical characteristic function test of independence with normal marginals

M. Bilodeau[a,*], P. Lafaye de Micheaux[b]

[a] *Département de Mathématiques et de Statistique, Centre de Recherches Mathématiques, Université de Montréal, C.P. 6128, Succursale Centre-ville, Montréal Que., Canada H3C 3J7*

[b] *Université Pierre Mendès France, Laboratoire de Statistique et Analyse de Données (LabSAD), BP 47/F-38040 Grenoble Cedex 9, France*

Received 30 December 2002

Abstract

This paper proposes a semi-parametric test of independence (or serial independence) between marginal vectors each of which is normally distributed but without assuming the joint normality of these marginal vectors. The test statistic is a Cramér–von Mises functional of a process defined from the empirical characteristic function. This process is defined similarly as the process of Ghoudi et al. [J. Multivariate Anal. 79 (2001) 191] built from the empirical distribution function and used to test for independence between univariate marginal variables. The test statistic can be represented as a *V*-statistic. It is consistent to detect any form of dependence. The weak convergence of the process is derived. The asymptotic distribution of the Cramér–von Mises functionals is approximated by the Cornish–Fisher expansion using a recursive formula for cumulants and inversion of the characteristic function with numerical evaluation of the eigenvalues. The test statistic is finally compared with Wilks statistic for testing the parametric hypothesis of independence in the one-way MANOVA model with random effects.

AMS 1991 subject classification: 62H15; 62M99; 62E20; 60F05

Keywords: Characteristic function; Independence; Multivariate analysis; Serial independence; Stochastic processes

* Corresponding author.

E-mail addresses: bilodeau@dms.umontreal.ca (M. Bilodeau), pierre.lafaye-de-micheaux@upmf-grenoble.fr (P. Lafaye de Micheaux).

0047-259X/$ - see front matter
doi:10.1016/j.jmva.2004.08.011

1. Introduction

Different characterizations have led to various tests of independence. Let $p \geqslant 1$ be a fixed integer. Consider a partitioned random vector $\boldsymbol{\epsilon} = \left(\boldsymbol{\epsilon}^{(1)}, \ldots, \boldsymbol{\epsilon}^{(p)}\right)$ made up of p q-dimensional subvectors and a corresponding partitioned $\boldsymbol{t} = \left(\boldsymbol{t}^{(1)}, \ldots, \boldsymbol{t}^{(p)}\right)$, for any fixed vector $\boldsymbol{t}$. Independence of the subvectors may be characterized with the joint distribution function or characteristic function as

$$K_p(\boldsymbol{t}) = \prod_{k=1}^{p} K^{(k)}(\boldsymbol{t}^{(k)}), \tag{1.1}$$

$$C_p(\boldsymbol{t}) = \prod_{k=1}^{p} C^{(k)}(\boldsymbol{t}^{(k)}), \tag{1.2}$$

where K_p and C_p are, respectively, the joint distribution function and joint characteristic function. The marginal versions are $K^{(k)}$ and $C^{(k)}$ for $k = 1, \ldots, p$. In the univariate setting ($q = 1$) Blum et al. [4] proposed an empirical process based on (1.1), whereas Csörgő [10] defined a similar process based on (1.2). Feuerverger [19] proposed an empirical characteristic function version of the Blum et al. [4] test statistic. He pointed out difficulties with dimensions above 2.

Recently, in the univariate setting, Ghoudi et al. [21] introduced a new process based on a characterization of independence which is now presented. This characterization for $p = 3$ is implicit in the early paper of Blum et al. [4]. For any $A \subset I_p = \{1, \ldots, p\}$ and any $\boldsymbol{t} \in \mathbb{R}^p$, let

$$\mu_A(\boldsymbol{t}) = \sum_{B \subset A} (-1)^{|A \setminus B|} K_p(\boldsymbol{t}_B) \prod_{j \in A \setminus B} K^{(j)}(t^{(j)}).$$

The notation $|A|$ stands for cardinality of the set A and the convention $\prod_{\emptyset} = 1$ is adopted. The vector $\boldsymbol{t}_B$ is used to make a selection of components of $\boldsymbol{t}$ according to the set B,

$$(\boldsymbol{t}_B)^{(i)} = \begin{cases} t^{(i)}, & i \in B; \\ \infty, & i \in I_p \setminus B. \end{cases}$$

Independence can be characterized as follows: $\boldsymbol{\epsilon}^{(1)}, \ldots, \boldsymbol{\epsilon}^{(p)}$ are independent if and only if $\mu_A \equiv 0$, for all $A \subset I_p$ satisfying $|A| > 1$. This characterization was also given previously in a slightly different form in Deheuvels [15]. A Cramér–von Mises functional of each process in a decomposition of the empirical dependence process, obtained originally by Deheuvels [14], led them to a non-parametric test of independence in the non-serial and serial situations. The interest of this decomposition resides in the mutual asymptotic independence of all the processes in the decomposition and the simple form of the covariance which is expressed as a product of covariance functions of the Brownian bridge.

In the multivariate setting ($q \geqslant 1$), the present paper proposes tests of independence, built from a process relying on a similar independence characterization based on characteristic functions, when subvectors or marginals are normally distributed. Namely, the marginals