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Some large-sample results on a modified Monte Carlo integration method

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Abstract

The large-sample properties of the modified Monte Carlo integration method with locally antithetic variates proposed by Haber (Math. Comput. 21 (1967) 388) are provided under the mild assumption that the integrand is a C^1 function. Moreover, the asymptotic distribution of the modified Monte Carlo estimator is obtained under the same condition. Furthermore, we propose a consistent variance estimation method, which avoids the replicated procedure considered by Haber (1967) which reduces the efficiency of this integration technique. On the basis of the achieved results, in addition to the integration framework, the method may be conveniently applied in the environmental sampling setting.

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1. Introduction

Suppose that f is a $L^2(K)$ integrable function, where $K = [0, 1]^d$. Hence, let us consider the estimation of the integral

$$I = \int_K f(u)\,\mathrm{d}u. \tag{1}$$

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As is well known, the estimation of (1) may be performed by using the Monte Carlo (MC) quadrature method (see e.g. Evans and Swartz, 2000), which is adopted in order to avoid the “curse of dimensionality” phenomenon. However, the MC estimator solely displays a $O(n^{-1})$ variance rate for each d and hence, more refined MC integration methods have been proposed. In this setting, the so-called randomized quasi-Monte Carlo method (RQMC) is effective for the estimation of (1) (L’Ecuyer and Lemieux, 2002; Owen, 1998). Indeed, if the mixed derivatives of order d of the integrand satisfy the Lipschiz condition, the RQMC estimator has a $O((\log n)^{d-1}n^{-3})$ variance rate (Owen, 1998). However, the method is rather complex to implement and no results are known for non-smooth integrands. Accordingly, it may be helpful to consider a simpler estimator which performs well in the case of less smooth integrands or in simulation studies where repeated integration is required, i.e. when a simple and fast integration procedure is needed.

An easy and efficient integration method is the modified Monte Carlo (MMC) method proposed by Haber (1966, 1967). The basic MMC method may produce an estimator with $O(n^{-1-2/d})$ variance rate if the Lipschitz condition for the integrand is assumed (Barabesi and Marcheselli, 2003) and it is always preferable to the basic MC integration for each n and d (Haber, 1966). Barabesi and Marcheselli (2003) have also provided the large-sample normality of the estimator and a consistent variance estimation method. The MMC method with locally antithetic variates proposed by Haber (1967) is even more effective than the basic MMC method, since it produces an estimator with a $O(n^{-1-4/d})$ variance rate if the mixed derivatives of second order of the integrand are continuous (Haber, 1967). Therefore, for $d \leqslant 2$ the MMC with locally antithetic variates is preferable to RQMC and is generally competitive for small d. It is worth noting that the expression “locally antithetic variates” may be misleading since the method is truly a mix of stratified sampling and systematic sampling. However, the method was formerly named by Haber in his seminal paper of 1967 and it is usually referred to the statistical literature with this name. Finally, besides the basic integration framework, the MMC method has interesting applications to environmental designs (Barabesi and Pisani, 2004; Barabesi, 2003).

In this paper, we achieve Haber’s (1967) results under milder conditions—i.e. by assuming the Lipschiz condition on the integrand’s gradient—as well as the large-sample normality of the estimator. In addition, we propose a consistent and simple variance estimation. This technique avoids using Haber’s (1967) replicated procedure, which reduces the efficiency of the MMC method with locally antithetic variates.

2. The large-sample results

Let us suppose that $n = m^d$. Haber (1966) introduced the basic MMC estimator for (1)

$$\tilde{I}_n = \frac{1}{n} \sum_{j \in J_n} f(V_j),$$

where $j = (j_1, \ldots, j_d)^{\mathrm{T}}$ and J_n is the d-Cartesian product of the set of the first m integers, i.e. $J_n = \{1, \ldots, m\}^d$. Moreover, the V_j’s are n independent d-dimensional random vectors, whose components are in turn independent uniform random variables $\mathscr{U}((j_i - 1)/m,\, j_i/m)$