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Confidence sets for the maximizers of intensity functions

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Abstract

Identifying times or time intervals when the intensity function of a Poisson process is maximal is important in a variety of practical problems, for instance in traffic control or with planning issues involving customer arrivals or accident occurrences. For this purpose, we propose confidence sets that are intuitive and easy to obtain, which makes them practicable for a quick exploratory data analysis. They may also be used in the context of mode estimation for probability densities. In the current literature, confidence sets for the mode are based on the assumption of an—at least locally—unique mode. In contrast, our approach retains the coverage probability even if the underlying intensity or density has a flat top. We even allow the intensity to be constant in the extreme.
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1. Introduction

Various practical phenomena lead to realizations of inhomogeneous Poisson processes. A frequent goal is to obtain information concerning the unknown intensity function. For this purpose nonparametric estimates for intensities have been proposed by several authors, including Diggle (1985), Diggle and Marron (1988), and Leadbetter and Wold (1983).

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Applications have been considered, e.g., to data concerning the occurrence of explosive volcanism (Solow, 1991) and to coal mining disaster occurrences (Diggle and Marron, 1988). In these applications, a strong dependence of the estimates on the bandwidth has been observed which makes them difficult to interpret. It is therefore of interest to find out whether interesting properties of the estimated intensity are mere artifacts or actually present. For this purpose, Cowling et al. (1996) proposed methods to construct uniform confidence bands. They investigate the classical coal mining disaster data (see Barnard, 1953; Cox and Lewis, 1966), and use the fact that the lower confidence bound at the year 1870 exceeds the upper confidence limit after 1900 to confirm a decrease in disaster occurrence after 1900. Instead of this indirect approach, we focus on the direct construction of interval estimates for the point(s) where the intensity is maximal. Possible applications of such interval estimates include the identification of times where customer arrivals, accident occurrences or traffic intensities are typically maximal. An illustrating application to ship arrival data at Keelung harbor (Taiwan) will be discussed in more detail in Section 5.

As detailed below, our considered problem is related to that of mode estimation for probability densities. Mode estimation has been considered by several authors. Here, we cite only some of the relevant literature. For unimodal densities, Parzen (1962), Chernoff (1964), Romano (1988), and Grund and Hall (1995) investigate the use of mode estimates based on kernel density estimates. More recently, mode trees have been proposed by Minnotte and Scott (1993) as visual tools for identifying possible modes. Minnotte et al. (1998) considered “mode forests” as a possible robustification of mode trees. Finally, tests for unimodality or, more generally, concerning the number of modes have been investigated for instance by Hartigan and Hartigan (1985), Silverman (1981), Mammen et al. (1992) as well as by Cheng and Hall (1998). While all proposed methods assume either one or a finite number of modes, our proposed confidence sets also work with densities and intensities that have a flat top. The assumption of a finite number of extremal points is frequently reasonable in the density estimation context, but it is unclear whether the assumption is justified in the context of Poisson processes, where periods of constant intensity often seem plausible. Therefore, we do not assume unimodality and even admit the possibility of a constant intensity in the extreme case. With multiple modes, our proposed confidence sets identify those that are global extremes.

We will now state our problem more formally. Assume that we observe an inhomogeneous Poisson process $X(t)$ on a time interval $[0, m]$ whose intensity function $\lambda^*(t)$ has period 1. While periodic intensities are encountered in many situations (think e.g. of daily, monthly or yearly fluctuations), the assumption of periodicity is not essential in the derivation of asymptotic coverage probabilities. It is easy to verify that our asymptotic results can also be obtained by considering $\lambda_l^* = l\mu$ for some density μ and letting $l \to \infty$, an approach chosen e.g. in Cowling et al. (1996).

Our goal is to construct confidence sets C for the point(s) where the intensity function λ^* is maximal using the $N = X(m)$ observed jump points of $X(t)$ on $[0, m]$. Let S denote the argmax set of λ^*, i.e.

$$S = \left\{ t \in [0,1] : \lambda^*(t) = \sup_s \lambda^*(s) \right\}.$$

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