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The butterfly effect under STV

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Abstract

This note presents an example of the sometimes chaotic character of the single transferable vote (STV) that is both somewhat simpler, and even more striking, than previous examples, and it offers several comments about the practical and theoretical implications of this feature of STV.

Keywords: STV; Chaos; Butterfly effect

> The 'Butterfly Effect,' or more technically the 'sensitive dependence on initial conditions,' is the essence of chaos.[1]

More than twenty years ago, Michael Dummett (1984, p. 280) observed that the single transferable vote (STV) method of election can operate in an arbitrary fashion "in which a small change in the ballot papers returned by a few voters will make a radical alteration in the overall outcome." Dummett returned even more emphatically to this point in his more recent book (1997, p. 142, emphasis added):

> The assessment process of STV … may, however, be said to be *quasi-chaotic*, in that small changes at the initial stage may be magnified into huge changes at later stages, because they cause different candidates to be eliminated, and that in turn may result in a big variation in the allocation of votes at subsequent stages, owing to the differing redistributions of votes from one candidate and from another.

Dummett (1997, pp. 143–149) also provided an example involving eight candidates contesting four seats before an electorate of 99,995 voters. More recently, Geller (2005, p. 267) picked up on Dummett's example, also invoked the concept of chaos, and specifically referred to the "butterfly effect." [2]

In this brief note, I present an example of the butterfly effect under STV that is somewhat simpler, and even more striking, than Dummett's example. I also offer comments about its practical and theoretical implications.

1. An example

Consider the following example. Seven candidates (A, B, C, D, E, F, and G) are competing before an

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[1] Michael Cross, http://www.cmp.caltech.edu/~mcc/chaos_new/Lorenz.html.

[2] To remedy the quasi-chaotic character of STV, Dummett recommends what he calls the "Quota/Borda system"(which combines elements of proportional representation with Borda scores), and Geller recommends STV with Borda elimination.

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electorate of 1001 voters for three seats. The quota for election is therefore 251 (and the residual 248 votes will be "wasted").

Voter preferences on the morning of the election are given by ballot profile 1 shown in Table 1A. As shown in Table 1B, the vote transfer process under ballot profile 1 results in the election of C, F, and G.

But before ballots are cast, a butterfly flaps its wings—or, more accurately, two butterflies flap their wings.[3] Before getting to their polling places, two voters slightly modify their preferences with respect to A and B (both losing candidates under ballot profile 1). While both voters initially ranked A first and B second, they now both rank B first and A second, but they make no other changes in their rankings, and none of the other 999 voters makes any change whatsoever in his or her preferences. The resulting ballot profile is shown in Table 2A (with column 2* excluded).

What would we expect the electoral consequence of this slight change in the ballot profile to be? Most likely, that it would have no effect at all on the winning vs. losing status of any of the candidates. But if it were to produce a change, we would most likely expect that B (now ranked higher by two voters) would convert from losing to winning status and one of the previously winning candidates C, F, or G would convert to losing status to make room for B among the winners.

In fact, this change in the ballot profile does *not* convert B from losing to winning status. But, in all other respects, it has a maximally profound impact—that is, *it converts the winning vs. losing status of every other candidate* (see Table 2B). C, F, and G now all lose and D and E now win, despite the fact that no voter has changed his or her ranking of any of these candidates. Moreover, A also converts from losing to winning status, despite the fact that the only voters who have changed their preferences moved A down in their rankings. The example therefore also illustrates STV's by the now well-known "monotonicity" problem. Note further that the two voters who changed their ballots continue to prefer the now losing F and G to the now winning D and E. Indeed, it can be checked that both voters can push A down to the very bottom of their ballot ranking (as shown in column 2*, producing ballot profile 2*) without affecting the sequence of vote transfers in Table 2B (since A is elected before their ballots transfer), in which event they prefer *all* the old winners to *all* the new winners elected as a result of their own slight ballot changes.

[3] Two butterflies are needed to preclude ties.

Table 1A
Ballot profile 1

144	125		160	145	153	126	148
144	27	98	160	145	153	126	148
A	B	B	C	D	E	F	G
B	C	F	G	G	C	A	F
C	G	A	F	F		B	D
G		D		A		C	A
		E		E			E

The key feature of the example is that candidates B and F are virtually tied with the fewest first preferences; the flapping (or not) of butterfly wings determines who gets eliminated first. Before the two voters change their preferences, B is eliminated and F picks up most of the ballots transferred from B, thereby surviving second-round elimination also, which sets up a cascade of transfers that leads to the election of C, E, and F. But after the two voters change their preferences, F is eliminated at the outset, F's ballots are transferred to (and elect) A, and a quite different cascade of transfers leads to the election of a completely different set of winners.

It should be noted that this example is in no way affected by any of the practical problems associated

Table 1B
Vote transfers under ballot profile 1

	A	B	C	D	E	F	G
(1)	144	[125]	160	145	153	126	148
(2)	[144]	–	187	145	153	224	148
(3)	–	–	*331*	145	153	224	148
(4)	–	–	**251**	[145]	153	224	228
(5)	–	–	**251**	–	153	224	*373*
(6)	–	–	**251**	–	153	*346*	**251**
(7)	–	–	**251**	–	248	**251**	**251**

(1) First preferences on all ballots are tallied. No candidate meets quota, so the weakest candidate is eliminated. By a single ballot, B has the fewest votes and is eliminated. (2) Following the second preferences indicated, 27 of B's ballots transfer to C and 98 to F. It remains true that no candidate meets the quota, so the next weakest candidate is eliminated. Votes having now transferred from B to F, A is the weakest remaining candidate and is eliminated. (3) Given that candidate B has been eliminated, all of A's 144 ballots transfer to C, who now meets quota and is elected. (4) Candidate C's surplus ballots transfer on the basis of lower preferences. The highest ranked remaining candidate on all 331 of C's (original and transferred) ballots is G, so all 80 surplus ballots transfer to G, but G still does not meet quota. The weakest candidate D is eliminated. (5) Following the second preference on all of D's ballots, all of D's 145 ballots transfer to G, who now meets quota and is elected. (6) Candidate G's surplus ballots transfer on the basis of lower preferences. The highest ranked remaining candidate on all 373 of G's (original and transferred) ballots is F, so all 122 surplus ballots transfer to F, who now meets quota and is elected. (7) Three candidates having been elected, the vote-counting process terminates with candidate E (after a final transfer of surplus ballots from F) holding the residual 248 "wasted votes."