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Some results on two-level regular designs with general minimum lower-order confounding

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A B S T R A C T

Zhang et al. (2008) introduced an aliased effect-number pattern (AENP) for two-level regular designs and proposed a general minimum lower-order confounding (GMC) criterion for choosing optimal designs. By using a finite projective geometric formulation, Zhang and Mukerjee (2009a) characterized GMC designs via complementary designs for general s-level case, and to find GMC designs, for some special cases they proved a result that a design T can have GMC only if $\overline{T}$ is contained in a specific flat. In this paper, we first generalize the result to general cases for $s=2$. Then, we prove that, for any given n and m, a GMC design minimizes A_3, the first term of the wordlengh pattern of regular 2^{n-m} designs. Furthermore, we find out the unique optimal confounding structure between main effects and two-factor interactions, and prove that minimizing A_3 is a sufficient and necessary condition for a regular design to have the structure.

1. Introduction

The effect hierarchy principle (EHP) is one of the most important principles in experimental design (Wu and Hamada, 2000). It states that, lower-order effects are likely more important than higher-order ones and effects with the same order are likely equally important. Therefore, in order to choose good designs, one should minimize the confounding between the lower-order effects. Towards this aim, quite a few criteria were proposed in the last decades, among them the minimum aberration (MA), clear effects (CE) and maximum estimation capacity (MEC) criteria (respectively proposed by Fries and Hunter, 1980; Wu and Chen, 1992; Sun, 1993) are mostly received. For a detailed summary, we refer to Mukerjee and Wu (2006).

Recently, by introducing an aliased effect-number pattern (AENP), Zhang, Li, Zhao, Ai (2008) (hereafter called ZLZA), proposed a general minimum lower-order confounding (GMC) criterion. We recall some concepts and results of them here first.

For a regular 2^{n-m} design T, let ${}_i^{\#}C_j^{(k)}(T)$ (write it as ${}_i^{\#}C_j^{(k)}$ for short) denote the number of the i th-order effects that is aliased with k j th-order effects, where k is called the severe degree of an i th-order effect being aliased with j th-order effects. The set $\{{}_i^{\#}C_j^{(k)} : i,j=0,1,\ldots,n, k=0,1,\ldots,K_j\}$, where $K_j=\binom{n}{j}$, is called the AENP of design T. Then, according to the EHP, with ignoring some trivial terms they rank the elements of the AENP in the following sequence:

$$ {}^{\#}C=({}_1^{\#}C_2, {}_2^{\#}C_2, {}_1^{\#}C_3, {}_2^{\#}C_3, {}_3^{\#}C_2, {}_3^{\#}C_3, \ldots), \tag{1} $$

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where

$$_i^{\#}C_j=(_i^{\#}C_j^{(0)},{}_i^{\#}C_j^{(1)},\ldots,{}_i^{\#}C_j^{(K_j)}), \tag{2}$$

and call the design that sequentially maximizes the components of the sequence (1) a GMC design. It can be seen that, comparing with other criteria, the GMC criterion chooses optimal designs with a more elaborate and explicit manner.

ZLZA showed that each of the MA, CE, and MEC criteria sequentially minimizes or maximizes the terms that are determined by the AENP. Thus the MA criterion sequentially minimizes the terms $A_3, A_4,\ldots$ in the wordlength pattern (WLP) (A_3, A_4, $A_5,\ldots$) and the A_i's are determined by the AENP as $A_i={}_i^{\#}C_0^{(1)}, i=3,4,5,\ldots$. Similarly, the CE criterion sequentially minimizes the terms c_1 and c_2, denoting the numbers of clear main effects and two-factor interactions (2fi's), and these terms are determined by the AENP as $c_1={}_1^{\#}C_2^{(0)}$ and $c_2={}_2^{\#}C_2^{(0)}-{}_1^{\#}C_2^{(1)}$. The MEC criterion can also be obtained by a specified function of the AENP, see Theorem 4 of ZLZA.

Therefore, in this sense, if we call the theory with using the AENP a general minimum confounding (also abbreviate it as GMC, for saving notation) theory, then it can hold the promise of the development of a unified theory encapsulating all the existing criteria, including GMC criterion.

Up to now, there are quite a few succedent works on the GMC theory, such as the characterization of GMC designs with prime or prime power s-level by Zhang and Mukerjee (2009a), the investigation of optimal blocking with GMC by Zhang and Mukerjee (2009b) and Zhang et al. (submitted for publication), and a series of construction theories of two-level GMC designs by Li et al. (in press), Zhang and Cheng (2010) and Cheng and Zhang (2010). Among them, Zhang and Mukerjee (2009a) not only characterized GMC designs via complementary designs for general s-level case by using a finite projective geometric formulation, but also, to find GMC designs, for some special cases they proved a result that a design T can have GMC only if $\overline{T}$ is contained in a specific flat.

In addition, in the passed investigations, people have noted that, any good design under the EHP should minimize the first term A_3 in the WLP of regular designs, however such a guess is still not proved. Thus, the immediate questions are raised: Does any GMC design minimize A_3 and what a special confounding structure between main effects and 2fi's does a design possessing the minimum A_3 lead to?

In this paper, we first extend the result about the relation between a GMC design and its complementary set given in Zhang and Mukerjee (2009a) for some special cases to general cases for $s=2$. Then, we answer the above questions: we prove that, for any given n and m, a GMC 2^{n-m} design must minimize A_3; furthermore, we find out the unique optimal confounding structure between main effects and 2fi's of regular designs, and prove that minimizing A_3 is a sufficient and necessary condition for a regular design to have the optimal structure.

2. Properties of GMC designs related to complementary sets

2.1. Some preliminary materials

To study further properties of GMC designs, here we firstly emphasize some preliminary material, including some definitions and notations, given in Zhang and Mukerjee (2009a). In this paper, we only discuss two-level case with resolution III or higher. An important tool for studying 2^{n-m} designs is the finite projective geometry. An $(r-1)$-dimensional finite projective geometry over $GF(2)$ is denoted by $PG(r-1,2)$. We use a typical pencil $b=(b_1,\ldots,b_n)'$ to denote a factorial effect, which is an $n\times 1$ nonnull vector with elements from $GF(2)$. A pencil b represents a main effect if it has exactly one nonzero element and a 2fi if it has exactly two nonzero elements and so on. Let P denote the set of $L_{n-m}=2^{n-m}-1$ different points of the finite projective geometry $PG(n-m-1,2)$. For any nonempty subset T of P, let $V(T)$ denote the matrix given by the points of T as columns. It is well known that, a design with resolution III or higher is equivalent to a set T of n points of $PG(n-m-1,2)$ with $V(T)$ having full row rank $n-m$ and satisfying the conditions (a)–(c) in Theorem 2.7.1 in Mukerjee and Wu (2006). Let $\overline{T}$ denote the complementary set of T in $PG(n-m-1,2)$ and f denote the cardinality of $\overline{T}$. Obviously, we have

$$f=2^{n-m}-1-n. \tag{3}$$

Any ω $(1\le\omega\le n-m)$ linearly independent points of $PG(n-m-1,2)$ can span a $(\omega-1)$–flat with cardinality L_ω.

Consider a point γ of P and a nonempty subset T. Let q denote the cardinality of T and Ω_{iq} the set of all $q\times 1$ vectors with i nonzero elements. In order to develop a theory for the GMC criterion in terms of complementary sets, for $i\ge 1$, Zhang and Mukerjee (2009a) defined

$$B_i(T,\gamma)=\#\{\lambda:\lambda\in\Omega_{iq},V(T)\lambda=\gamma\}, \tag{4}$$

where # denotes the cardinality of a set. They also showed that

$$_1^{\#}C_2^{(k)}=\#\{\gamma:\gamma\in T,\tfrac{1}{2}(n-f-1)+B_2(\overline{T},\gamma)=k\}, \tag{5}$$

$$_1^{\#}C_2^{(k)}=(k+1)[\#\{\gamma:\gamma\in T,\tfrac{1}{2}(n-f-1)+B_2(\overline{T},\gamma)=k+1\}+\#\{\gamma:\gamma\in \overline{T},\tfrac{1}{2}(n-f+1)+B_2(\overline{T},\gamma)=k+1\}]. \tag{6}$$