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Monitoring and diagnosing dependent process steps using VSI control charts

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ABSTRACT

The paper proposes the variables sampling interval (VSI) scheme to monitor the means and the variances in two dependent process steps. The performance of the considered VSI control charts is measured by the adjusted average time to signal derived by a Markov chain approach. An example of the process control for the metallic film thickness of the computer connectors system shows the application and performance of the proposed VSI control charts in detecting shifts. Furthermore, the performance of the VSI control charts and the fixed sampling interval control charts are compared via the numerical analysis results. These demonstrate that the former is much faster in detecting shifts. Whenever quality engineers cannot specify the values of variable sampling intervals, the optimal VSI control charts are recommended. Furthermore, the impacts of misusing Shewhart charts to monitoring the process mean and variance in the second process step are also investigated.

1. Introduction

Control charts are important tools in statistical quality control. They are used to effectively monitor whether a process is in-control or out-of-control. Shewhart (1931) developed the $\overline{X}$ control charts which are easy to implement and have been widely used for industrial processes. However, even though Shewhart $\overline{X}$ control charts are used to monitor a process by taking samples of equal size at a fixed sampling interval (FSI), it is usually slow in signaling small to moderate shifts in the process mean. Consequently, several alternatives have been developed in recent years to improve the performance of $\overline{X}$ control chart. One of the useful approaches to improve the detection ability is to use a variable sampling interval (VSI) and/or a variable sample size (VSS) control chart instead of the traditional FSI and/or fixed sample size (FSS). If there were some indication that a process parameter may be changed, the next sampling interval should be shorter and/or the next sample should be larger. Otherwise the next sampling interval should be longer and/or the next sample should be smaller.

The properties of the $\overline{X}$ and $\overline{X}-R$ charts with VSIs were studied by Reynolds et al. (1988), Reynolds and Arnold (1989), Chengular et al. (1989), Amin and Miller (1993), Baxley (1996) and Reynolds et al. (1996). Tagaras (1998) reviewed the literature on adaptive control charts. Reynolds and Stoumbos (2001) discussed the properties of VSI X and MR control charts for individual observations. These papers show that most work on developing VSI control charts had aimed to solve the problem of monitoring process mean.

However, these papers assume that there is only a single process step whereas many products are currently produced with several dependent process steps. Consequently, it is not appropriate to monitor these process steps by utilizing a control chart for each individual process step. Zhang (1984) proposed the simple cause-selecting control chart to control the specific

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quality in the current process by adjusting the effect of the incoming quality variable (X) on outgoing quality variable (Y) since X and Y are dependent. The cause-selecting values (e) are Y minus the effect of X, and the cause-selecting control chart is constructed accordingly. Wade and Woodall (1993) reviewed and analyzed the cause-selecting control chart and examined the Hotelling T^2 control chart. In their opinion the cause-selecting control chart outperforms the Hotelling T^2 control chart. However, the properties of the VSI control charts used to control mean and variance in two dependent process steps have not yet been addressed. Therefore, we need to study the performance of the joint VSI control charts on two dependent process steps. In this paper, we propose the joint VSI $Z_{\overline{X}}-Z_{S^2}$ and $Z_{\overline{e}}-Z_{S_e^2}$ control charts with variable sampling intervals to control the mean and the variance in two dependent process steps. In the next section, the performance of the VSI $Z_{\overline{X}}-Z_{S^2}$ and $Z_{\overline{e}}-Z_{S_e^2}$ control charts is measured by the adjusted average time to signal (AATS) using a Markov chain approach. Finally, we illustrate the application of the proposed control charts by an example of the metallic film thickness of the computer connectors system. We also compare the performance between the VSI $Z_{\overline{X}}-Z_{S^2}$ and $Z_{\overline{e}}-Z_{S_e^2}$ control charts and FSI $Z_{\overline{X}}-Z_{S^2}$ and $Z_{\overline{e}}-Z_{S_e^2}$ control charts. If the variable sampling intervals cannot be specified the optimal VSI $Z_{\overline{X}}-Z_{S^2}$ and $Z_{\overline{e}}-Z_{S_e^2}$ control charts should be used. The impacts of misusing $Z_{\overline{Y}}-Z_{S_y^2}$ charts to monitoring the process mean and variance in the second step are also investigated.

2. Description of the joint VSI $Z_{\overline{X}}-Z_{S^2}$ and $Z_{\overline{e}}-Z_{S_e^2}$ charts

Consider a process with two dependent process steps controlled by the joint VSI $Z_{\overline{X}}-Z_{S^2}$ and $Z_{\overline{e}}-Z_{S_e^2}$ control charts. Let X be the measurable incoming quality variable on the first process step. Assume further that this process starts in a state of statistical control, that is, X follows a normal distribution with the mean at its target value, μ_X, and the standard deviation at its target value σ_X. Let Y be the measurable outgoing quality characteristic of interest for the second process step and follow a normal distribution conditional on X. Since the second process step is affected by the first process step, then following Wade and Woodall (1993), the relationship between Y and X is generally expressed as

$$Y_i|X_i = f(X_i)+\varepsilon_i, \quad i=1,2,3,\ldots,m \tag{1}$$

where $\varepsilon_i \sim \mathrm{NID}(0,\sigma^2)$. Let Y represent $Y|X$. If the function $f(X_i)$ is known, the values of the standardized error term $\varepsilon_i^*=((Y_i-f(X_i))/\sigma)$ are called the cause-selecting values since they are the values of Y_i adjusted for the effects of X_i. In practice, the true function $f(X_i)$ is usually unknown and thus must be estimated using the data of the initial m observations. Thus the estimate for $f(X_i)$ will be $\hat{Y}_i$. The residuals, $e_i=Y_i-\hat{Y}_i \sim \mathrm{NID}(0,\sigma_e^2)$. The standardized residuals $e_i^*=((Y_i-\hat{Y}_i)/\sigma_e)$ are called the cause-selecting values. The X chart is thus constructed to monitor the mean of X_i on the first step, and the e chart is constructed to monitor the mean of e_i on the second step.

However, in our study the chosen sample size is not one and the rational samples of size n are taken from the two dependent process steps. The sample data are plotted to obtain a sample profile and then establish the reference line of Y and X (see Kim et al., 2003). To monitor the mean and variance of X on the first step the $\overline{X}-S^2$ charts should be constructed, and to monitor the mean and variance of e on the second step the $\overline{e}-S_e^2$ charts should be constructed. The $\overline{X}-S^2$ charts and $\overline{e}-S_e^2$ charts are called cause-selecting control charts.

For engineers to use the control charts easily, the sample means and sample variances are standardized as follows:

$$\begin{aligned} Z_{\overline{X_i}} &= \frac{\overline{X_i}-\mu_X}{\sigma_X/\sqrt{n}} \sim N(0,1), \quad Z_{S_i^2} = \frac{(n-1)S_i^2}{\sigma_x^2} \sim \chi^2(n-1) \\ Z_{\overline{e_i}} &= \frac{\overline{e_i}}{\sigma_e/\sqrt{n}} \sim N(0,1), \quad Z_{S_{e_i}^2} = \frac{(n-1)S_{e_i}^2}{\sigma_e^2} \sim \chi^2(n-1) \end{aligned} \tag{2}$$

where $\overline{X}_i=(\sum_{j=1}^n X_{ij})/n$, $S_i^2=(\sum_{j=1}^n (X_{ij}-\overline{X_i})^2)/(n-1)$, $\overline{e_i}=(\sum_{j=1}^n e_{ij})/(n)$ and $S_{e_i}^2=(\sum_{j=1}^n (e_{ij}-\overline{e_i})^2)/(n-1)$ $i=1, 2, 3,\ldots,m$.

Without losing generalization, assume that once a special cause occurs it affects the X-variable with probability ν and the functional relationship (or e-variable) with probability $1-\nu$. That is, the mean of X_{ij} shifts from μ_X to $\mu_X+\delta_1\sigma_X$ ($\delta_1 \neq 0$) and the standard deviation shifts from σ_X to $\delta_2\sigma_X$ ($\delta_2>1$) with probability v, and the mean of the e_{ij} shifts from 0 to δ_3($\delta_3 \neq 0$) and the standard deviation shifts from σ_e to $\delta_4\sigma_e$ ($\delta_4>1$) with probability $1-\nu$. The out-of-control distribution of X_{ij} and/or e_{ij} will be adjusted to in-control state, once at least one true signal is obtained from the proposed control charts. Let T_{sc} be the time until the occurrence of a special cause and be following an exponential distribution of the form

$$f(t)=\lambda\exp(-\lambda t), \quad t>0 \tag{3}$$

where $1/\lambda$ is the mean time that the process remains in a state of statistical control.

An in-control state analysis for the joint VSI $Z_{\overline{X}}-Z_{S^2}$ and $Z_{\overline{e}}-Z_{S_e^2}$ control charts is performed since the shifts in the process mean and variance on step 1 and/or step 2 do not occur when the process is just starting, but occur at some time in the future. The standardized samples $z_{\overline{X_i}}-z_{S^2}$ and $z_{\overline{e}_i}-z_{S_e^2}$ are plotted on the joint VSI $Z_{\overline{X}}-Z_{S^2}$ and $Z_{\overline{e}}-Z_{S_e^2}$ control charts with warning limits of the form $\pm w_{\overline{X}}$, w_{S^2}, $\pm w_{\overline{e}}$ and $w_{S_e^2}$, and control limits of the form $\pm k_{\overline{X}}$, k_{S^2}, $\pm k_{\overline{e}}$ and $k_{S_e^2}$, respectively, where $0 \le w_{\overline{X}} < k_{\overline{X}}$, $0 \le w_{S^2} < k_{S^2}$, $0 \le w_{\overline{e}} < k_{\overline{e}}$ and $0 \le w_{S_e^2} < k_{S_e^2}$.

The search for the special cause and adjustment in the first process step is undertaken when the sample $z_{\overline{X_i}}$ falls outside the interval $(-k_{\overline{X}}, k_{\overline{X}})$ and/or when the z_{S^2} falls outside the interval $(0, k_{S^2})$, that is when the $Z_{\overline{X}}$ and/or z_{S^2} charts produce a signal.