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Checking nonlinear heteroscedastic time series models

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Abstract

A procedure for testing simultaneously, the parametric forms of the conditional mean and the conditional variance functions of a real-valued heteroscedastic time series model is proposed. The Wald test statistic is based on a vector whose components are suitable normalized sums of some weighted residual series. The test is consistent under some fixed alternatives. The local power under two sequences of local alternatives is studied. A LAN property for the parametric model of interest is also established. Experiment conducted shows that the test performs well on the examples tested.

MSC: 62M10; 62G10; 62F03

Keywords: Contiguity; Goodness of fit; Heteroscedastic models; LAN; Local alternatives; Time series

1. Introduction

With the recent development in non-parametric and semi-parametric time series, there is a growing interest in testing nonlinear time series models. A substantial part of the existing literature is concerned with tests based on comparing parametric versus non-parametric (Härdle and Mamen, 1993; Tjøstheim and Auestad, 1994; McKeague and Zhang, 1994; Hjellvik and Tjøstheim, 1995; Hjellvik et al., 1998). Another part proposes tests based on marked empirical type processes (An and Cheng, 1991; Chen and An, 1997; Ngatchou-Wandji and Laı¨b, 1998; Koul and Stute, 1999; Ngatchou-Wandji, 2002). There

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are, however, many other papers relevant to this problem, such as Brockett et al. (1988) and Tjøstheim (1996).

Because of the technical difficulties encountered in the derivation of tests for models of general order, most of the existing tests are devoted to testing the nonlinearity of first-order models. Moreover, this nonlinearity is usually tested through testing individually, the nonlinearity of the conditional mean function and/or that of the conditional variance function. For the purpose of generalization, and because of a possible gain in time, it may be interesting to test simultaneously these functions when they are associated with models of order larger than one, and then consider testing them individually only if the null hypothesis of interest is rejected. In time series analysis, the problem of testing simultaneously many functions has already been considered e.g., in Li (1999) and Chen and Fan (1999), where some consistent tests for time-series econometric models are derived for mixing data.

The present paper deals with testing jointly, the parametric forms of the conditional mean and the conditional variance functions of the real-valued models

$$X_{i+1} = T(\mathbf{X}_i) + V(\mathbf{X}_i)\varepsilon_{i+1}, \quad i \geqslant 0, \tag{1}$$

where the sequence of random variables (rv's) $\{X_i : i \geqslant 0\}$ is stationary and ergodic; the random vectors $\mathbf{X}_i = (X_{i-q}, X_{i-q+1}, \ldots, X_i)$, $i \geqslant 0$, have an unknown common distribution function $\mathbf{F}$ and q is a given non-negative integer; the ε_i's are standard independent and identically distributed (iid) rvs; the real-valued functions $T(\cdot)$ and $V(\cdot)$ are unknown. In other words, this paper is concerned with testing whether the couple of functions $(T(\cdot), V(\cdot))$ belongs to a given class of parametric functions or not. More precisely, let K and P be positive integers and

$$\mathscr{M} = \left\{(m(\rho;\cdot), \sigma(\theta;\cdot)), (\rho', \theta')' \in \Theta_0 \times \tilde{\Theta}_0\right\},$$

where $\Theta_0 \subset \mathbb{R}^K$ and $\tilde{\Theta}_0 \subset \mathbb{R}^P$ are compact, and each of the functions $m(\cdot)$ and $\sigma(\cdot)$ has a known form. For a sample of length $n+1$, we derive a test for $\tilde{\mathrm{H}}_0[(T(\cdot), V(\cdot)) \in \mathscr{M}]$ against $\tilde{\mathrm{H}}_1[(T(\cdot), V(\cdot)) \notin \mathscr{M}]$. One can remark that the hypothesis $\tilde{\mathrm{H}}_0$ is equivalent to $\mathrm{H}_0[(T(\cdot), V(\cdot)) = (m(\rho_0;\cdot), \sigma(\theta_0;\cdot))]$ for some $(\rho_0', \theta_0')' \in \Theta_0 \times \tilde{\Theta}_0$, while the alternative hypothesis $\tilde{\mathrm{H}}_1$ is equivalent to $\mathrm{H}_1[(T(\cdot), V(\cdot)) \neq (m(\rho_0;\cdot), \sigma(\theta_0;\cdot))]$. To derive a feasible and consistent test for this problem, we first observe that under H_0, the conditional mean and the conditional variance functions of (1) are almostsurely (a.s.) defined by $E\{(X_1 - m(\rho_0; \mathbf{X}_0))|\mathbf{X}_0\} = 0$ and $E[\{(X_1 - m(\rho_0; \mathbf{X}_0))^2 - \sigma^2(\theta_0; \mathbf{X}_0)\}|\mathbf{X}_0] = 0$. Thus, for any Borelian sets $\mathbf{A}$ and $\mathbf{B}$ of $\mathbb{R}^q$, each of $n^{-1}\sum_{i=1}^n (X_{i+1} - m(\rho_0; \mathbf{X}_i))I_{\mathbf{A}}(\mathbf{X}_i)$ and $n^{-1}\sum_{i=1}^n \{(X_{i+1} - m(\rho_0; \mathbf{X}_i))^2 - \sigma^2(\theta_0; \mathbf{X}_i)\}I_{\mathbf{B}}(\mathbf{X}_i)$ converges a.s. to 0, where $I_\Omega(\cdot)$ denotes the indicator function. We then consider the standardized processes

$$\hat{A}_n = n^{-1/2} \sum_{i=1}^{n} (X_{i+1} - m(\rho_0; \mathbf{X}_i))I_{\mathbf{A}}(\mathbf{X}_i), \tag{2}$$

$$\hat{B}_n = n^{-1/2} \sum_{i=1}^{n} \{(X_{i+1} - m(\rho_0; \mathbf{X}_i))^2 - \sigma^2(\theta_0; \mathbf{X}_i)\}I_{\mathbf{B}}(\mathbf{X}_i). \tag{3}$$