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Combining asymptotically normal tests: case studies in comparison of two groups

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Abstract

When several candidate tests are available for a given testing problem, and each has nice properties with respect to different criteria such as efficiency and robustness, it is desirable to combine them. We discuss various combined tests based on asymptotically normal tests. When the means of two standardized tests under contiguous alternatives are close, we show that the maximum of the two tests appears to have an overall best performance compared with other forms of combined tests considered, and that it retains most power compared with the better one of the two tests combined. As an application, for testing zero location shift between two groups, we studied the normal, Wilcoxon, median tests and their combined tests. Because of their structural differences, the joint convergence and the asymptotic correlation of the tests are not easily derived from the usual asymptotic arguments of the tests. We developed a novel application of martingale theory to obtain the asymptotic correlations and their estimators. Simulation studies were also performed to examine the small sample properties of these combined tests. Finally we illustrate the methods by a real data example.

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1. Introduction

In the studies of mapping disease genes, the genetic models are often used in testing whether the markers are linked to the latent disease genes (Pawlowitzki et al., 1997). While an approximately correct specification of these models achieves the most power, a misspecification could result in a considerable loss of power (Clerget-Darpoux et al., 1986). In the hope of reducing power loss under model misspecification, Abreu et al. (1999) considered two χ^2 test statistics that are derived under two vastly different genetic models, and used the maximum between the two as a new test statistic. Of course, the 95th percentile of the χ^2 distribution is no longer valid as the critical value for the new test. Based on some empirical evidence, they suggested adjusting the critical value by adding 0.3 to it. In survival analysis, Fleming and Harrington (1991) considered the maximum of a class of weighted logrank test statistics as a versatile test. Given a family of asymptotically most powerful tests, Gastwirth (1966, 1985) considered a maxmin efficiency robust test that is often of a linear combination form. In general, parametric tests are efficient under a specific distribution, while nonparametric tests are robust under a broad family of distributions. To achieve both robustness and efficiency, combining tests seems a natural and logical approach.

Motivated by the aforementioned examples, we investigate the use of maximum as well as some other common choices in forming new tests based on several candidate tests that are asymptotically normal. We refer to these new tests as combination tests. At issue is how to adjust the critical value of the combination tests properly, whether there are any gains in power for using the combination tests, and how the maximum compares with other alternative combined tests. We consider four combined tests for the standardized versions of the test statistics: their average, the average of their absolute values, the maximum of their absolute values, and the χ^2 test statistic, and study their asymptotic power. In the case studies of testing zero location shift in two groups, we study the performance of the normal, Wilcoxon, median tests and their combined tests, both asymptotically and in finite sample simulations.

Technically, critical values of the various combined tests can be obtained based on the estimated asymptotic correlation coefficient matrix. Since in applications the tests being combined often are derived very differently, it is not straightforward to establish the joint limiting distribution and to obtain the parameters in the joint limiting distribution. For testing zero location shift in two groups, we use the martingale and counting processes techniques to address those technical difficulties.

The remainder of this paper is organized as follows. Section 2 defines the four combined tests, Section 3 compares the power of these tests under contiguous alternatives. Section 4 presents the case studies for testing zero location shift between two groups, and Section 5 shows a real data example, followed by the conclusions in Section 6. All the proofs are arranged in the Appendices.

2. Combined tests

Let $T_1, \ldots, T_k$ be k candidate test statistics for the null hypothesis $H_0{:}\gamma = 0$ of the parameter γ against the two-sided alternative. Suppose that, under H_0, $(T_1, \ldots, T_k)$ has a limiting multivariate normal distribution with zero mean, variances $\sigma_1^2, \ldots, \sigma_k^2$ and correlation coefficients ρ_{ij}, $i, j = 1, \ldots, k$. Under a sequence of contiguous alternative hypotheses, we