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Robust slippage rank tests for k location parameters in the presence of gross errors

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Abstract

A robust slippage test problem of k location parameters in the presence of gross errors is formulated from the point of view of Huber's robust test theory. Under an asymptotic model of the robust slippage test problem an asymptotic level α slippage rank test based on k linear rank statistics is constructed by applying majorization methods and its asymptotic minimum power is evaluated by applying weak majorization methods. It is also shown that the slippage rank test is asymptotically unbiased.

MSC: 62G10; 62G20; 62G35

Keywords: Robust slippage test problem of location; Slippage rank test; Gross error; Huber's robust test theory; Asymptotic maximum size; Asymptotic minimum power; Shrinking neighborhood; Majorization; Weak majorization; Schur-convex set; Schur-concave function

1. Introduction

An important class of multiple decision problems is that of slippage problems. Because slippage problems have some symmetric structure of a null hypothesis and k alternative hypotheses such as permutation equivariance, they have been treated in a manner similar to hypothesis testing. Slippage problems were first introduced by Mosteller (1948) as a problem of testing homogeneity of k populations against k slippage alternatives that exactly one of the k populations is different. Paulson (1952), who treated the

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slippage problem of normal mean, was the first to formulate the problem satisfactorily. Since then, many contributions have been made to such slippage test problems. Among them, there are Traux (1953), Kudo (1956), Doornbos and Prins (1958), Karlin and Traux (1960), Hall and Kudo (1968a), Hall et al. (1968b), Kimura and Kudo (1974), Kakiuchi and Kimura (1975), Kakiuchi et al. (1975), Kimura (1984a,1988a) and so forth.

The robust slippage test problem was proposed by Kimura (1984b) from the point of view of robust test theory of Huber (1965), Huber and Strassen (1973) and Rieder (1977). He formulated it as a problem of testing a neighborhood of distributions against k neighborhoods of distributions and derived a robust slippage version of the Neyman–Pearson's lemma. Kimura (1988b) considered a robust asymptotic slippage test problem which was formulated by applying Rieder's (1978) asymptotic model with gross error neighborhoods shrinking at the rate of order $n^{-1/2}$. By using majorization and weak majorization inequalities, he constructed asymptotic level α slippage tests and gave lower bounds for their asymptotic minimum powers.

On the other hand, Kimura and Kakiuchi (1989) and Kakiuchi and Kimura (1995) developed some majorization methods on hyperplanes and studied their applications to various robust tests for approximate equality in the parametric setup. Kakiuchi and Kimura (2001) proposed a test problem of k-sample approximate equality in the nonparametric setup, which can be regarded as a generalization of Rieder's (1981) problem for the two-sample case. By using the majorization methods, under an asymptotic model with shrinking gross error neighborhoods they derived lower and upper bounds for the limiting probability that a random vector of k-sample rank statistics takes in a Schur convex set. As their applications, they constructed asymptotic level α rank tests for the k-sample approximate equality and obtained lower bounds for their asymptotic minimum powers, which were used for discussions of asymptotic relative efficiency.

The purpose of this paper is (1) to give a formulation of robust asymptotic slippage test problems of k location parameters in the presence of gross errors, (2) to construct asymptotic level α slippage rank tests and (3) to derive lower bounds for their asymptotic minimum powers. To do this, we make use of Kakiuchi and Kimura's (2001) results and weak majorization inequalities.

In Section 2, we formulate a robust slippage test problem of k location parameters in the presence of gross errors. In Section 3, we introduce a class of slippage rank tests based on certain score generating functions which are permutation equivariant. In Section 4, we give an asymptotic model with shrinking gross error neighborhoods for the robust slippage test problem of k location parameters. In Section 5, we collect auxiliary results which are used to establish main results of this paper. In Section 6, by applying Kakiuchi and Kimura's (2001) majorization methods we construct asymptotic level α slippage rank tests and derive lower bounds for their asymptotic minimum powers. It is also shown that the constructed slippage rank tests are asymptotically unbiased. Finally, we recommend a score generating function.

2. The k-sample robust slippage problem of location

Let $\mathscr{X}$ be the extended real line, $\mathscr{B}$ the σ-field of Borel subsets of $\mathscr{X}$ and $\mathscr{M}$ the set of all probability measures on $\mathscr{B}$. Let $\mathscr{M}_c$ denote the subset of $\mathscr{M}$ that corresponds