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Discrete M-robust designs for regression models

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Abstract

M-robust designs are defined and constructed for misspecified linear regression models with possibly autocorrelated errors on a discrete design space. These designs minimize the mean-squared errors if linear regression models are correct with uncorrelated errors, subject to two robust constraints which control the change of the bias and the change of variance under model departures. Simulated annealing algorithm is applied to construct M-robust designs. Examples are given to show M-robust designs and compare them with minimax robust designs.

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Keywords: Annealing algorithm; Change of bias function; Change of variance function; Discrete design; Moving average process; Penalty function

1. Introduction

Robust designs have been studied for regression models against departures both in the regression response and in the assumption of uncorrelated errors. Commonly used approach is the minimax approach, and the resulting designs are called minimax robust designs. These designs usually minimize the maximum loss of the mean-squared error (MSE), and the maximum is taken over all possible departures from the regression model. Recently Wiens and Zhou (1997) introduced an infinitesimal approach to derive robust designs, and V-robust, B-robust and M-robust designs are defined and studied. The main idea is to minimize the MSE at the ideal model (without any model departures), subject to some constraints which guarantee small changes in the MSE if there are small departures from model assumptions. Change of variance function and change of bias function are defined and used in those constraints. All minimax

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design problems and V-robust, B-robust and M-robust design problems are hard to solve. Wiens and Zhou (1996, 1997, 1999) provided analytical solutions to some regression models, but there still remain open problems deriving analytical solutions for robust designs.

In this paper, we explore M-robust designs for approximately linear regression models with possibly correlated errors and provide numerical solutions. We consider a finite (discrete) design space, which consists of finite many points, to construct M-robust designs numerically. The resulting designs are called discrete M-robust designs. With a finite design space, the optimization problem associated with robust designs can be simplified considerably both analytically and numerically. This sheds light on the construction of robust designs. For example, Fang and Wiens (2000) constructed minimax robust designs for approximately linear models with possible heteroscedastic errors. A simulated annealing algorithm is developed to search for the discrete designs. Zhou (2001) studied discrete minimax robust designs with a modified simulated annealing algorithm.

In order to use simulated annealing algorithms to search for minimax robust designs, we need to derive analytically the maximum loss of the MSE over all possible model departures. The annealing algorithms only do the minimization step. The model departures considered in Zhou (2001) include the departure from the regression response and the departure from the assumption of uncorrelated errors. In particular, the errors follow a moving average process. The maximum loss of the MSE over the first and second-order moving average processes (MA(1) and MA(2)) are obtained analytically. Guidelines to obtain the maximum loss over higher-order moving average processes are given but can be difficult to work out. M-robust designs provide an alternative way to construct robust designs and avoid the difficulty of maximization. Examples show that minimax designs and M-robust designs are similar for MA(1) and MA(2), but M-robust designs can be easily computed for MA(q) ($q \geqslant 3$).

The rest of the paper is organized as follows. In Section 2, we define M-robust designs for regression models with departures in the response and in the assumption of uncorrelated errors. In particular, the change of bias function and the change of variance function are introduced and discussed. M-robust design problem is a constrained optimization problem. In Section 3, we transform the constrained optimization problem into a unconstrained one, then apply the annealing algorithm in Zhou (2001) to construct M-robust designs. Examples are given to show M-robust designs and compare them with minimax designs. We conclude with a few remarks in Section 4.

2. M-robust designs

Consider a regression model with departures in the response and in the assumption of uncorrelated errors:

$$y_i = \mathbf{z}^{\mathrm{T}}(\mathbf{x}_i) \cdot \theta + f(\mathbf{x}_i) + \varepsilon_i, \quad i = 1, \ldots, n, \tag{2.1}$$

where y_i are responses of variable Y at regressor level $\mathbf{x}_i \in \mathscr{R}^q$, vector $\mathbf{z}(\mathbf{x}) \in \mathscr{R}^p$ is a specified function of $\mathbf{x}$, $\theta \in \mathscr{R}^p$ is the regression parameter, function $f(\mathbf{x})$ is