Statistics and Probability Letters 83 (2013) 2248–2255

Contents lists available at ScienceDirect

Statistics and Probability Letters

journal homepage: www.elsevier.com/locate/stapro

Precise large deviations of aggregate claims in a risk model with regression-type size-dependence[☆]

Xiuchun Bi, Shuguang Zhang [*]

Department of Statistics and Finance, School of Management, University of Science and Technology of China, Hefei, Anhui 230026, PR China

ARTICLE INFO

Article history:
Received 8 January 2013
Received in revised form 9 June 2013
Accepted 9 June 2013
Available online 19 June 2013

MSC:
62P05
62E05
60K30
60K05

Keywords:
Aggregate claims
Web Markov skeleton process
Consistent variation
Dependence
Large deviations

ABSTRACT

In this note, we further extend the renewal risk model and introduce a more practical regression-type dependence structure between inter-arrival times and claim sizes, which is described by the framework of a web Markov skeleton process. We investigate large deviations of the aggregate claims, and, for a case of heavy-tailed claims, we obtain a precise large-deviation formula of the aggregate claims under some assumption about regression-type size-dependence, which is consistent with existing ones in the literature.

1. Introduction

Throughout this note, let $Y_n, n = 1, 2, \dots$ and $T_n, n = 1, 2, \dots$ be claim sizes and inter-arrival times, respectively. We assume that $Y_n, n = 1, 2, \dots$ form a sequence of independent and identically distributed (i.i.d.) random variables, and that $T_n, n = 1, 2, \dots$ form a sequence of identically distributed nonnegative random variables, but not necessarily independent. Denote by Y and T the generic random variables of them, respectively. The claim arrival times are $\tau_n = \sum_{k=1}^{n} T_k, n \geq 1$, with $\tau_0 = 0$. Then the number of claims by time $t \geq 0$ is $N_t = \sup_n\{n \geq 1 : \tau_n \leq t\}$, which forms a generalized renewal counting process. In this way, the aggregate amount of claims is of the form

$$S_t = \sum_{k=1}^{N_t} Y_k, \quad S_0 = Y_0 = 0,\ t \geq 0. \tag{1.1}$$

If $T_n,\ n \geq 1$ are mutually independent and independent of $\{Y_n, n \geq 1\}$, then we obtain the standard renewal risk model, which has been playing a fundamental role in classical and modern risk theory since it was introduced by Sparre Andersen in the middle of the last century. If $(Y_n, T_n), n \geq 1$ are i.i.d. copies of (Y, T), with dependent components Y and T, then

☆ Supported by the National Basic Research Program of China (Project No. 973-2007CB814901).
* Corresponding author. Tel.: +86 551 63607294.
E-mail addresses: xcbi@mail.ustc.edu.cn (X. Bi), sgzhang@ustc.edu.cn, bxcustc@gmail.com (S. Zhang).

0167-7152/$ – see front matter Crown Copyright © 2013 Published by Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.spl.2013.06.009

we obtain the non-standard renewal risk model, which was first proposed by Albrecher and Teugels (2006) and further studied by Boudreault et al. (2006), Cossette et al. (2008), and Badescu et al. (2009), among many others. Recently, Asimit and Badescu (2010) introduced a general dependence structure for (Y, T), via the conditional tail probability of Y given T; see also Li et al. (2010). In particular, Chen and Yuen (2012) considered the conditional tail probability of T given Y, and, under the assumption that there is a random variable $T^* > 0$ such that

$$\Pr(T > t|Y > x) \leq \Pr(T^* > t) \tag{1.2}$$

holds for $t \geq 0$ and x large enough, they studied the large deviations of the aggregate amount of heavy-tailed claim sizes and obtained a precise large-deviation formula.

$\Pr(T > t|Y > x)$ means that the waiting time for a large claim is dependent on the claim size. But, practically, the waiting time for a large claim depends not only on the next claim size, but also on the previous claim sizes, which has not been studied yet.

Recently, a new class of stochastic processes, called web Markov skeleton processes (WMSPs for short), arising from information retrieval on the Web, has been found to be very useful in various natural and social sciences, such as finance, queueing theory, insurance, and other related fields. See Liu et al. (2011, 2012) for the notion of WMSPs.

Intuitively, a WMSP is a jump process and also a Markov skeleton process such that, for the given information of its skeleton, the time slots between jumps are conditionally independent of each other. The dynamics of a WMSP can be described as follows:

$$X_0 \stackrel{\tilde{T}_0}{\rightarrow} X_1 \stackrel{\tilde{T}_1}{\rightarrow} \cdots X_n \stackrel{\tilde{T}_n}{\rightarrow} \cdots, \tag{1.3}$$

where $\{X_n, n \geq 0\}$ is a Markov chain, and $\{\tilde{T}_n, n \geq 0\}$ is the set of time slots between adjacent jumps.

The framework of WMSP covers various classes of processes, for instance, discrete-time Markov chains, time-homogeneous continuous-time Markov processes (Q-processes), semi-Markov processes, and so on. It also contains some new classes of processes, which are important either for applications or theoretical study. In particular, if a WMSP satisfies

$$\Pr(\tilde{T}_n \leq t|\mathcal{F}^X) = \Pr(\tilde{T}_n \leq t|X_n, X_{n+1}), \quad \forall\, n \geq 0, t \geq 0, \tag{1.4}$$

we call it a semi-Markov process, where $\mathcal{F}^X$ is generated by $X = \{X_n, n \geq 0\}$.

Motivated by (1.4), we consider the aggregate amount of claims (1.1) with the following dependence structure between $\{Y_n, n \geq 1\}$ and $\{T_n,\ n \geq 1\}$:

$$\textbf{DS}: \begin{array}{l} \Pr(T_n > t|\mathcal{F}^Y) = \Pr(T_n > t|Y_n, Y_{n-1}), \quad \forall\, n \geq 2, \\ \Pr(T_1 > t|\mathcal{F}^Y) = \Pr(T_1 > t|Y_1), \end{array} \tag{1.5}$$

where $\mathcal{F}^Y$ is generated by $\{Y_n, n \geq 1\}$. Equipped with other modeling factors (such as initial surplus, premium incomes, etc.) and incorporated with some economic factors (such as interests, dividends, taxes, returns on investments, etc.), this model provides a good mechanism to describe a non-life insurance business, e.g., automobile insurance. Furthermore, it allows applications in various areas.

The precise large deviation of random sums has been extensively investigated in many literatures since it was initiated by Klüppelberg and Mikosch (1997), for example, Ng et al. (2003, 2004), Kaas and Tang (2005), Wang and Wang (2007), Lin (2008), Baltrūnas et al. (2008), Liu (2009), Li et al. (2010) and Chen and Yuen (2012), among many others. But the corresponding result under the above-mentioned dependence structure has not been considered in the literature. In this note, we will study the precise large deviation of (1.1) under dependence structure (1.5).

Here is a brief outline of this note. Section 2 gives the main result after some preliminaries. Section 3 gives some lemmas, and Section 4 presents the proofs of the main result.

2. Preliminaries and main result

For convenience, we introduce the following notation that is used throughout this paper.

- *For two positive functions $a(x)$ and $b(x)$, we write*

$$a(x) \sim b(x) \quad \text{if } \lim_{x\to\infty} \frac{a(x)}{b(x)} = 1,$$

$$a(x) \lesssim b(x) \quad \text{if } \limsup_{x\to\infty} \frac{a(x)}{b(x)} \leq 1.$$

- *For two positive bivariate functions $a(\cdot,\cdot)$ and $b(\cdot,\cdot)$, we say that $a(x,t) \lesssim b(x,t)$, as $t \to \infty$, holds uniformly in $x \in \Delta_t \neq \emptyset$ if*

$$\limsup_{t\to\infty} \sup_{x\in\Delta_t} \frac{a(x,t)}{b(x,t)} \leq 1.$$

- *For a distribution function $F(x)$ with finite mean $\mu > 0$, set $\overline{F}(x) \equiv 1 - F(x)$ as its corresponding survival function.*