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A prophet inequality for  $L^2$ -martingales

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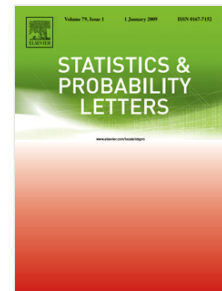
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**Abstract**

Let  $k = (k_n)_{n \geq 1}$  be the sequence given by the conditions  $k_1 = 0$  and  $k_{n+1} = (1 + k_n^2)/2$ ,  $n \geq 1$ . We prove that for any  $L^2$ -martingale  $X = (X_1, X_2, \dots, X_n)$  we have

$$\mathbb{E} \max_{1 \leq k \leq n} X_k \leq \sup_{\tau} \mathbb{E} X_{\tau} + k_n \max_{1 \leq k \leq n} \sqrt{\text{Var } X_k},$$

where the supremum on the right is taken over all stopping times  $\tau$  of  $X$  which are bounded by  $n$ . Furthermore, it is shown that for each  $n$ , the constant  $k_n$  is the best possible.

*Keywords:* Prophet inequality, optimal stopping, martingale

*2010 MSC:* 60G40, 60E15, 60G42

**1. Introduction**

Let  $X = (X_1, X_2, \dots, X_n)$  be a sequence of random variables on some common probability space  $(\Omega, \mathcal{F}, \mathbb{P})$  and let  $X_n^* = \max_{1 \leq k \leq n} X_k$  denote the one-sided maximal function of  $X$ . Furthermore, let  $M_n = \mathbb{E} X_n^*$  and  $V_n = \sup_{\tau} \mathbb{E} X_{\tau}$ , where the latter supremum is taken over all stopping times  $\tau$  of  $X$  (i.e., all  $\tau$  adapted to the natural filtration of  $X$ ), which do not exceed  $n$ . In the literature, comparisons between the numbers  $M_n$  and  $V_n$  (under various additional assumptions on  $X$ ) have been called “prophet inequalities”. This is due to the natural identification of  $M_n$  with the optimal expected return of a prophet or a player endowed with complete foresight (up to time  $n$ ). On the other hand,  $V_n$  can be regarded as an optimal expected return, on the time interval  $\{1, 2, \dots, n\}$ , of a player who knows only past and present, but not the future. Prophet inequalities play a distinguished role in the theory of optimal stopping and have been studied intensively by many mathematicians. The literature on the subject is very large, and it would be impossible to review it here. Thus we will content ourselves with a few examples, and refer the interested reader to the expository paper by Hill and Kertz (1992), and, for some more recent work, to Assaf et al. (2002), Allaart and Monticino

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