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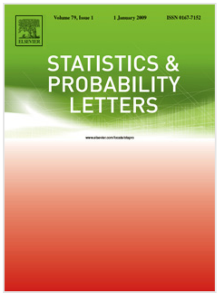

A prophet inequality for L^2-martingales

Adam Osękowski

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A prophet inequality for L^2-martingales

Adam Osękowski

Department of Mathematics, Informatics and Mechanics
University of Warsaw
Banacha 2, 02-097 Warsaw
Poland

Abstract

Let $k = (k_n)_{n\geq 1}$ be the sequence given by the conditions $k_1 = 0$ and $k_{n+1} = (1 + k_n^2)/2$, $n \geq 1$. We prove that for any L^2-martingale $X = (X_1, X_2, \ldots, X_n)$ we have

$$\mathbb{E} \max_{1\leq k\leq n} X_k \leq \sup_\tau \mathbb{E}X_\tau + k_n \max_{1\leq k\leq n} \sqrt{\operatorname{Var} X_k},$$

where the supremum on the right is taken over all stopping times τ of X which are bounded by n. Furthermore, it is shown that for each n, the constant k_n is the best possible.

Keywords: Prophet inequality, optimal stopping, martingale
2010 MSC: 60G40, 60E15, 60G42

1. Introduction

Let $X = (X_1, X_2, \ldots, X_n)$ be a sequence of random variables on some common probability space $(\Omega, \mathcal{F}, \mathbb{P})$ and let $X_n^* = \max_{1\leq k\leq n} X_k$ denote the one-sided maximal function of X. Furthermore, let $M_n = \mathbb{E}X_n^*$ and $V_n = \sup_\tau \mathbb{E}X_\tau$, where the latter supremum is taken over all stopping times τ of X (i.e., all τ adapted to the natural filtration of X), which do not exceed n. In the literature, comparisons between the numbers M_n and V_n (under various additional assumptions on X) have been called "prophet inequalities". This is due to the natural identification of M_n with the optimal expected return of a prophet or a player endowed with complete foresight (up to time n). On the other hand, V_n can be regarded as an optimal expected return, on the time interval $\{1, 2, \ldots, n\}$, of a player who knows only past and present, but not the future. Prophet inequalities play a distinguished role in the theory of optimal stopping and have been studied intensively by many mathematicians. The literature on the subject is very large, and it would be impossible to review it here. Thus we will content ourselves with a few examples, and refer the interested reader to the expository paper by Hill and Kertz (1992), and, for some more recent work, to Assaf et al. (2002), Allaart and Monticino

Email address: ados@mimuw.edu.pl (Adam Osękowski)
URL: www.mimuw.edu.pl/~ados (Adam Osękowski)