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Inequality constrained ridge regression estimator

Selma Toker *, Gülesen Üstündağ Şiray, Selahattin Kaçıranlar

Çukurova University, Faculty of Science and Letters, Department of Statistics, Adana, 01330, Turkey

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ABSTRACT

We carry out the idea of inequality constrained least squares (ICLS) estimation of Liew (1976) to the inequality constrained ridge regression (ICRR) estimation. We propose ICRR estimator by reducing the primal–dual relation to the fundamental problem of Dantzig–Cottle (1967,1974) with Lemke (1962) algorithm. Furthermore, we conduct a Monte Carlo experiment.

1. Introduction

An inequality constrained model is

$$y = X\beta + \varepsilon \tag{1}$$

$$A\beta \geq c \tag{2}$$

where y is an $n \times 1$ vector of observations on the dependent variable; X has a full column rank of $n \times p$ nonstochastic explanatory variables matrix; β is a $p \times 1$ vector of unknown parameters; A is an $m \times p$ fixed matrix; c is a vector of m components. ε is an $n \times 1$ vector of disturbances with expectation $E(\varepsilon) = 0$ and dispersion matrix $\mathrm{Var}(\varepsilon) = \sigma^2 I$.

The primal–dual relation of Dorn (1960) and Mangasarian (1962) is as follows:

$$\min z = f(\beta); \quad \text{subject to } A\beta \geq c, \ \beta \text{ unrestricted} \tag{3}$$

and

$$\max Q = f(\beta_0) - \beta_0' \bigtriangledown f(\beta_0) + \lambda' c; \quad \text{subject to } A'\lambda = \bigtriangledown f(\beta_0), \ \lambda \geq 0. \tag{4}$$

To introduce the least squares estimator with inequality restrictions on parameters, Liew (1976) followed Dorn (1960) and Mangasarian (1962) to introduce the primal–dual relation as:

* Corresponding author. Tel.: +90 03223386084.
E-mail address: stoker@cu.edu.tr (S. Toker).

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primal

$$\min_{\beta} z = \frac{1}{2}(y - X\beta)'(y - X\beta) \tag{5}$$

subject to

$$A\beta \geq c \text{ (or } A\beta - \upsilon = c), \quad \beta \text{ unrestricted} \tag{6}$$

where υ is a nonnegative m component surplus vector,

dual

$$\max_{\lambda} Q = \frac{1}{2}\left(y'y - \beta'X'X\beta\right) + c'\lambda \tag{7}$$

subject to

$$A'\lambda + X'y = (X'X)\beta, \quad \lambda \geq 0 \tag{8}$$

where λ is an m component dual vector and β is a solution to the primal problem. A solution can be obtained by reducing the primal–dual relation to the fundamental problem of Dantzig–Cottle (1967,1974) and calculating with Lemke (1962) or Dantzig–Cottle (1967,1974) algorithms as:

$$\upsilon = W\lambda + q \tag{9}$$

subject to

$$\upsilon'\lambda = 0, \quad \upsilon \geq 0 \text{ and } \lambda \geq 0 \tag{10}$$

where

$$W = A\left(X'X\right)^{-1}A', \qquad q = A\hat{\beta} - c \quad \text{and} \quad \hat{\beta} = \left(X'X\right)^{-1}X'y. \tag{11}$$

A sufficient condition of the solution requires the matrix $X'X$ to be positive definite and λ^*, υ^* are nonnegative complementary solutions of the problem. By replacing λ^* in Eq. (8), Liew (1976) obtained the ICLS estimator, β^*, as:

$$\beta^* = \left(X'X\right)^{-1}X'y + \left(X'X\right)^{-1}A'\lambda^*. \tag{12}$$

If the parameters of the model are not restricted, the λ^* becomes zero and the β^* becomes equal to the ordinary least squares (OLS) estimator ($\hat{\beta}$). If the parameters are constrained by equalities all the elements of the λ^* in the complementary solution become positive which implies $\upsilon^* = 0$ and

$$\lambda^* = -\left(A\left(X'X\right)^{-1}A'\right)^{-1}\left(A\hat{\beta} - c\right). \tag{13}$$

Thus, the β^* reduces to the equality constrained least squares estimator as:

$$\beta^* = \hat{\beta} + \left(X'X\right)^{-1}A'\left(A\left(X'X\right)^{-1}A'\right)^{-1}(c - A\hat{\beta}). \tag{14}$$

The untruncated variance–covariance matrix of β^* is defined as

$$V(\beta^*) = \sigma^2 M\left(X'X\right)^{-1}M' \tag{15}$$

where $M = \left(I + X'X\right)^{-1}\tilde{A}_2'M_2A$. $\left(\tilde{A}_1' \quad \tilde{A}_2'\right)$ is a rearranged columnwise A matrix such that $A'\lambda^* = \left(\tilde{A}_1' \quad \tilde{A}_2'\right)\begin{bmatrix}0\\ \lambda^0\end{bmatrix}$ and $\begin{bmatrix}\upsilon^0\\ \lambda^0\end{bmatrix} = \begin{bmatrix}M_1\\ M_2\end{bmatrix}q$ is an m component vector of the basic variables of

$$[I_1 - W_1]\begin{bmatrix}\upsilon^0\\ \lambda^0\end{bmatrix} + [I_2 - W_2]\begin{bmatrix}\upsilon^c\\ \lambda^c\end{bmatrix} = q \tag{16}$$

where $[I_1 - W_1]$ is the m by m optimal basis; $\begin{bmatrix}\upsilon^c\\ \lambda^c\end{bmatrix}$ and $[I_2 - W_2]$ are the nonbasic variables and the corresponding nonbasis.

Liew (1976) examined the asymptotic properties of the ICLS estimator for two different cases. In the first case where all true parameters are unbounded, namely $A\beta \gg c$, the existence of a sufficiently large sample $n \geq n_0$ such that the ICLS estimator, β_n^*, on such large sample reduces to the OLS estimator, $\hat{\beta}_n$, is proved by a theorem. In the second case, some parameters are unbounded while the others are bounded, i.e., $A_1\beta \gg c_1$ and $A_2\beta = c_2$. If this prior belief is correct, the existence of a sufficiently large sample $n \geq n_0$ such that the ICLS estimator, β_n^*, on such a large sample reduces to the equality constrained least squares estimator is proved in another theorem.