Statistics and Probability Letters 83 (2013) 2034–2038

Contents lists available at SciVerse ScienceDirect

Statistics and Probability Letters

journal homepage: www.elsevier.com/locate/stapro

Tail probability of avoiding Poisson traps for branching Brownian motion

Mehmet Öz, Mine Çağlar *

Department of Mathematics, Koç University, Istanbul, Turkey

ARTICLE INFO

Article history:
Received 15 October 2012
Received in revised form 16 May 2013
Accepted 17 May 2013
Available online 24 May 2013

Keywords:
Branching process
Random environment
Poisson traps
Tail probability

ABSTRACT

We consider a branching Brownian motion Z with exponential branching times and general offspring distribution evolving in $\mathbb{R}^d$, where Poisson traps are present. A Poisson trap configuration with radius a is defined to be the random subset K of $\mathbb{R}^d$ given by $K = \bigcup_{x_i \in \text{supp}(M)} \bar{B}(x_i, a)$, where M is a Poisson random measure on $\mathbb{B}(\mathbb{R}^d)$ with constant trap intensity. Survival up to time t is defined to be the event $\{T > t\}$ with $T = \inf\{s \geq 0 : Z_s(K) > 0\}$ being the first trapping time. Following the work of Engländer (2000), Engländer and den Hollander (2003), where strictly dyadic branching is considered, we consider here a general offspring distribution for Z and settle the problem of survival asymptotics for the system.

1. Introduction

We consider a branching Brownian motion evolving in $\mathbb{R}^d$ (Ikeda et al., 1968; Dawson, 1993; Engländer, 2000). We start with a single particle at the origin at $t = 0$, performing Brownian motion for a random time which is distributed exponentially with constant parameter β. Then, the particle dies and simultaneously gives birth to a random number of particles distributed according to the offspring distribution μ, which is a probability measure on $\mathbb{N}$. Similarly, each offspring particle repeats the same procedure independently from all others, starting from the position of her parent. In this way, one obtains a measure-valued Markov process $Z = (Z_t)_{t\geq 0}$, which is a particle configuration on $\mathbb{R}^d$. We assume that $Z_0 = \delta_0$. The total mass process $|Z| = (|Z_t|)_{t\geq 0}$ is a continuous time μ-Galton–Watson process with branching rate β. We denote the extinction time of the process $|Z|$ by τ, which is formally defined as $\tau = \inf\{t \geq 0 : |Z_t| = 0\}$, where we use the convention that $\inf \emptyset = \infty$. We then denote the event of extinction of the process $|Z|$ by E, and define $E = \{\tau < \infty\}$. We use the term *non-extinction* for the event E^c, and save the term *survival* for the context of trapping. We let P be the probability for the process Z.

The branching Brownian motion is assumed to live in a random environment consisting of Poisson traps. Let M denote the Poisson random measure on $\mathbb{B}(\mathbb{R}^d)$ with mean measure $\nu = \upsilon \cdot \text{Leb}$, where Leb stands for the Lebesgue measure, and $\upsilon > 0$. A random trap configuration K with radius a on $\mathbb{R}^d$ is defined as

$$K = \bigcup_{x_i \in \text{supp}(M)} \bar{B}(x_i, a),$$

where $\bar{B}(x_i, a)$ is the closed ball of radius a centered at x_i. We let $\mathbb{P}$ be the probability for the Poisson traps, and $\mathbb{E}$ the corresponding expectation.

* Corresponding author. Tel.: +90 212 3381315.
E-mail address: mcaglar@ku.edu.tr (M. Çağlar).

0167-7152/$ – see front matter © 2013 Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.spl.2013.05.016

In this paper, we are interested in the probability that the branching system avoids the traps up to time t asymptotically, averaged over all trap configurations. Let T denote the first hitting time of K by the branching Brownian motion Z given by

$$T = \inf\{s \geq 0 : Z_s(K) > 0\}.$$

Survival up to time t is defined by the event $\{T > t\}$. Our object of interest is the annealed (averaged) survival probability of the system up to time t over all trap configurations, denoted by $\mathbb{E} \times P\{T > t\}$. We investigate its asymptotic behavior as $t \to \infty$ for various cases of the distribution μ. The goal is to exhaust all possibilities for μ.

2. Previous results on survival asymptotics

There are two major results for the survival asymptotics at hand, corresponding to two different deterministic instances of μ.

Donsker and Varadhan (1975) studied the asymptotic behavior of the volume of the Wiener sausage. The Wiener sausage with radius a is defined as

$$W_t^a = \bigcup_{0 \leq s \leq t} \bar{B}(W(s), a),$$

where W stands for the Wiener process in $\mathbb{R}^d$. Its asymptotic distribution is given by

$$\lim_{t\to\infty} t^{-\frac{d}{d+2}} \log E \exp(-v|W_t^a|) = -c(d, v), \tag{1}$$

where $c(d, v)$ is a constant depending on $v > 0$ and dimension d. Letting $|W_t^a|$ be the volume of the Wiener sausage, by using the definition of Poisson random measure and applying Fubini's theorem, one observes that $E \exp(-v|W_t^a|) = \mathbb{E} \times P\left\{\bar{T} > t\right\}$ for $t > 0$, where $\bar{T}$ is the analogue of T for a single Brownian particle. In view of this equivalence, the work of Sznitman (1998, Chapter 4) is an alternative proof of (1) in the context of Brownian survival among Poisson traps. Note that this solves the problem at hand for the case $\mu(1) = 1$, and the scaling is subexponential.

Engländer (2000), Engländer and den Hollander (2003) considered the problem for the strictly dyadic branching case, corresponding to $\mu(2) = 1$. The result is

$$\lim_{t\to\infty} \frac{1}{t} \log \mathbb{E} \times P\{T > t\} = -\beta \quad \text{for } d \geq 2 \tag{2}$$

and

$$\lim_{t\to\infty} \frac{1}{t} \log \mathbb{E} \times P\{T > t\} = \begin{cases} -\beta \dfrac{v}{\bar{v}} & \text{if } v \leq \bar{v} \\ -\beta & \text{if } v > \bar{v} \end{cases} \quad \text{for } d = 1 \tag{3}$$

where β is as before the branching rate of the process, and $\bar{v} = \frac{1}{2}\sqrt{\frac{\beta}{2}}$ is the critical Poisson intensity. We see that the scaling is now exponential as a result of branching, which was not present in the problem of Donsker and Varadhan (1975). Also, for $d \geq 2$, we see the dominating effect of branching over spatial motion, since the result only depends on a branching parameter.

3. Survival asymptotics for general offspring distribution

In this section, we consider all the possibilities for the offspring distribution μ. Observe that a continuous time μ-Galton–Watson process (with $\mu(1) \neq 1$) with lifespan parameter β is equal in law to the continuous time λ-Galton–Watson process with lifespan parameter $\beta(1 - \mu(1))$, where $\lambda(1) = 0$ and $\lambda(j) = \frac{\mu(j)}{1-\mu(1)}$ for $j \neq 1$. Therefore, we may and do assume henceforth that $\mu(1) = 0$ for all continuous time μ-Galton–Watson processes considered.

Now we state our main result. We note that the first two parts of the theorem follow directly from existing results and trivial comparisons, while the third part is a non-trivial result that requires the application of Lyons' result (1992) on the set of individuals with infinite line of descent.

Theorem 1. *Let Z be a BBM with offspring distribution μ and branching rate β. It is assumed that $\mu(1) = 0$ and $\beta > 0$. Let f be the generating function and τ be the extinction time for the underlying μ-Galton–Watson process, that is $f(x) = \sum_{j=0}^{\infty} \mu(j)x^j$, and $\tau = \inf\{t \geq 0 : |Z_t| = 0\}$, where we adopt the convention that $\inf \emptyset = \infty$. Then, the survival asymptotics for the problem of BBM among Poisson traps with constant trap intensity v is given as follows:*