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Stationary bootstrapping realized volatility

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ABSTRACT

First order asymptotic validity is established for stationary bootstrapping of the realized volatility. This enables us to construct a bootstrapping confidence interval for integrated volatility. A Monte-Carlo experiment shows that stationary bootstrapping confidence interval is also valid in a finite sample.

1. Introduction

Thanks to the availability of intraday high frequency financial asset data, realized volatilities are widely used as realizations of return volatilities. In recent years, statistical methods and probabilistic properties are actively investigated by many authors. Important results are summarized by, for example, Poon and Granger (2003), Barndorff-Nielsen and Shephard (2002), Pigorsch et al. (2012), and Hwang and Shin (2013).

Some authors studied distributions of the realized volatilities. Among others, Barndorff-Nielsen and Shephard (2002) and Barndorff-Nielsen et al. (2006) made significant contributions in theory of realized volatility by developing asymptotic theorems including the central limit theorems.

Recently Goncalves and Meddahi (2009) applied the i.i.d. bootstrapping for approximating distributions of realized volatilities. Dovonon et al. (2013) extended the i.i.d. bootstrapping to multivariate high frequency returns such as realized regression coefficients and realized covariances and correlations.

We note that the intraday return observations have dependence due to conditional heteroscedasticity. Therefore, block bootstrapping methods may be good alternatives to the i.i.d. bootstrapping because the dependence structure of the original observations is retained in resampled blocks. Among diverse block bootstrapping methods, the stationary bootstrapping of Politis and Romano (1994) is one of the most actively adopted ones in recent studies. See Hwang and Shin (2011), and Hwang and Shin (2012a) regarding nonparametric estimation; Swensen (2003) and Parker et al. (2006) regarding nonstationary time series analysis; Lahiri (1999), Goncalves and White (2002), Goncalves and de Jong (2003), Nordman (2009) and Hwang and Shin (2012b) for basic properties.

In this paper, the stationary bootstrap realized volatility is proposed and its asymptotic validity is verified by showing that the bootstrap version has the same limiting normal approximation as the realized volatility. Both weak consistency

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and strong consistency are established. The asymptotic result provides us a stationary bootstrap confidence interval for the integrated volatility. A Monte-Carlo simulation shows that the proposed confidence interval is also valid in a finite sample.

The remaining of the paper is organized as follows. In Section 2 we describe the setup and the existing theory that will be used in this work, and review the stationary bootstrap procedure. Main theoretical results are presented in Section 3 and a Monte-Carlo result is given in Section 4, while technical results and proofs are given in Section 5.

2. Preliminary setup

2.1. Existing asymptotic theory

Let $\{\log S_t : t \geq 0\}$ be the log-price process of a financial asset. We adopt the setup and notations of Barndorff-Nielsen et al. (2006) for the realized volatility to have central limit theorem (CLT) results because our bootstrapping results assume the CLT results. We have the following assumption.

(A1): The process $\{\log S_t : t \geq 0\}$ follows the continuous time process

$$d\log S_t = \mu_t dt + \sigma_t dW_t \tag{1}$$

where μ_t is the drift term, σ_t is a volatility process, and W_t is the standard Brownian motion.

We are interested in estimating the integrated volatility (IV) over a fixed time interval [0, 1] defined by

$$\mathrm{IV} = \int_0^1 \sigma_u^2\, du.$$

We have scaled the time unit so that the interval [0, 1] corresponds to the day of interest. A consistent estimator of the integrated volatility is the realized volatility (RV) defined by the sum of squared high frequency returns

$$\mathrm{RV}_{(h)} = \sum_{i=1}^{1/h} r_i^2 \tag{2}$$

where h is a real number such that $1/h$ is a positive integer number, and high frequency returns $r_i = \log S_{ih} - \log S_{(i-1)h}$ are measured over the period $[(i-1)h, ih]$ for $i = 1, 2, \ldots, 1/h$.

Barndorff-Nielsen and Shephard (2002) and Barndorff-Nielsen et al. (2006) established the following asymptotic normality under some assumptions including that the drift and volatility processes are jointly independent of $\{W_u : u \geq 0\}$

$$\frac{\sqrt{h^{-1}}(\mathrm{RV}_{(h)} - \mathrm{IV})}{\sqrt{2IQ}} \xrightarrow{\mathrm{d}} N(0, 1) \tag{3}$$

where IQ denotes the integrated quarticity, which is defined by $IQ = \int_0^1 \sigma_u^4\, du$. They established the CLT result (3) for the model (1) under (A1) and the following condition.

(A2): The volatility process σ_t satisfies

$$\sigma_t = \sigma_0 + \int_0^t a_u^* du + \int_0^t \sigma_u^* dW_u + \int_0^t v_u^* dV_u$$

where a_t^*, σ_t^*, v_t^* are the adopted cadlag processes with a_t^* being predictable and locally bounded and V_t is a Brownian motion independent of W_t.

This is the case where no jumps are allowed in the volatility, but it can be relaxed (see Barndorff-Nielsen et al., 2006, Assumption (H1) for a more general assumption on σ_t).

Also, according to Barndorff-Nielsen and Shephard (2002) and Barndorff-Nielsen et al. (2006), a consistent estimator of the integrated quarticity is the realized quarticity (RQ) defined by

$$\mathrm{RQ}_{(h)} = \frac{1}{3h} \sum_{i=1}^{1/h} r_i^4,$$

and thus we have

$$\frac{\sqrt{h^{-1}}(\mathrm{RV}_{(h)} - \mathrm{IV})}{\sqrt{2\mathrm{RQ}_{(h)}}} \xrightarrow{\mathrm{d}} N(0, 1).$$

2.2. Stationary bootstrap procedure

The stationary bootstrap method proposed by Politis and Romano (1994) is described. Recall that $r_i = \log S_{ih} - \log S_{(i-1)h}$ is the log-return for time between $(i-1)h$ and ih, $i = 1, \ldots, n$ and $n = 1/h$. Let $r_1, \ldots, r_n$ be observed. First we define