Statistics and Probability Letters 83 (2013) 2081–2087

Contents lists available at SciVerse ScienceDirect

Statistics and Probability Letters

journal homepage: www.elsevier.com/locate/stapro

On pairwise quasi-asymptotically independent random variables and their applications

Jinzhu Li *

School of Mathematical Science and LPMC, Nankai University, Tianjin 300071, PR China

ARTICLE INFO

Article history:
Received 29 December 2012
Received in revised form 21 May 2013
Accepted 22 May 2013
Available online 27 May 2013

MSC:
primary 62P05
secondary 62E10
91B30

Keywords:
Asymptotics
Dominated variation
Quasi-asymptotic independence
By-claims
Ruin probability

ABSTRACT

In this paper we obtain some novel results regarding pairwise (strong) quasi-asymptotically independent random variables with dominatedly varying tails. Our main concern lies in the asymptotics for constant and randomly weighted sums of such random variables. The obtained results are applied to study the ultimate ruin probability of a claim-dependent risk model.

1. Introduction and preliminaries

Throughout this paper, all limit relationships hold as $x \to \infty$ unless stated otherwise. For two positive functions $a(\cdot)$ and $b(\cdot)$, we write $a(x) \gtrsim b(x)$ or $b(x) \lesssim a(x)$ if $\liminf a(x)/b(x) \geq 1$, write $a(x) \sim b(x)$ if both $a(x) \gtrsim b(x)$ and $a(x) \lesssim b(x)$, and write $a(x) \asymp b(x)$ if both $a(x) = O(b(x))$ and $b(x) = O(a(x))$. To avoid trivialities, every real-valued rv (random variable) is assumed to be not only concentrated on $(-\infty, 0]$ unless stated otherwise.

Adopting the definition in Chen and Yuen (2009), real-valued rv's $X_1, X_2, \ldots$ with survival functions $\overline{F}_1, \overline{F}_2, \ldots$ are said to be pQAI (pairwise quasi-asymptotically independent) if, for any $i \neq j$,

$$\lim_{x\to\infty} \mathbb{P}(|X_i| \wedge X_j > x | X_i \vee X_j > x) = 0.$$

The above relation is obviously equivalent to

$$\lim_{x\to\infty} \frac{\mathbb{P}(X_i > x, X_j > x) + \mathbb{P}(X_i < -x, X_j > x)}{\overline{F}_i(x) + \overline{F}_j(x)} = 0.$$

Further, we say that $X_1, X_2, \ldots$ are pSQAI (pairwise strong quasi-asymptotically independent) if, for any $i \neq j$,

$$\lim_{x_i \wedge x_j\to\infty} \mathbb{P}(|X_i| > x_i | X_j > x_j) = 0,$$

* Tel.: +86 2223501233.
E-mail address: lijinzhu@nankai.edu.cn.

0167-7152/$ – see front matter © 2013 Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.spl.2013.05.023

which is equivalent to

$$\lim_{x_i \wedge x_j \to \infty} \frac{\mathbb{P}(X_i > x_i, X_j > x_j) + \mathbb{P}(X_i < -x_i, X_j > x_j)}{\overline{F}_j(x_j)} = 0;$$

see Geluk and Tang (2009) and Yang and Hashorva (2012). Clearly, if $X_1, X_2, \ldots$ are pSQAI then they are pQAI. Even the pSQAI case covers a wide range of dependence structures. Actually, $X_1, X_2, \ldots$ are pSQAI if they are, e.g., mutually independent, pairwise negatively dependent, or pairwise FGM (Farlie–Gumbel–Morgenstern) distributed, i.e., for any $i \neq j$,

$$\mathbb{P}(X_i \le x_i, X_j \le x_j) = F_i(x_i)F_j(x_j)\left(1 + a_{ij}\overline{F}_i(x_i)\overline{F}_j(x_j)\right), \tag{1.1}$$

where a_{ij} is a real number such that the right-hand side of (1.1) is a proper distribution. For original ideas and analogues of pQAI and pSQAI, we refer the reader to Maulik and Resnick (2004) and Resnick (1987, 2002), among others.

A survival function $\overline{F}$ is said to be dominatedly varying, denoted by $\overline{F} \in \mathscr{D}$, if

$$\overline{F}(xy) \asymp \overline{F}(x), \quad y > 0.$$

A smaller class than $\mathscr{D}$ is the class $\mathscr{C}$ of consistently varying functions. By definition, $\overline{F} \in \mathscr{C}$ if

$$\lim_{y \searrow 1} \liminf_{x\to\infty} \frac{\overline{F}(xy)}{\overline{F}(x)} = 1, \quad \text{or, equivalently,} \quad \lim_{y \nearrow 1} \limsup_{x\to\infty} \frac{\overline{F}(xy)}{\overline{F}(x)} = 1.$$

The most important subclass of the class $\mathscr{C}$ is the class ERV of extended regularly varying functions specified by

$$y^{-\beta} \le \liminf_{x\to\infty} \frac{\overline{F}(xy)}{\overline{F}(x)} \le \limsup_{x\to\infty} \frac{\overline{F}(xy)}{\overline{F}(x)} \le y^{-\alpha}, \quad y \ge 1, \tag{1.2}$$

for some $0 < \alpha \le \beta < \infty$. In this case, we write $\overline{F} \in \mathrm{ERV}(-\alpha, -\beta)$. Particularly, if $\alpha = \beta$ in relation (1.2), i.e.,

$$\overline{F}(xy) \sim y^{-\alpha}\overline{F}(x), \quad y > 0,$$

then $\overline{F}$ is said to be regularly varying, denoted by $\overline{F} \in \mathscr{R}_{-\alpha}$. In addition, $\overline{F}$ is said to be long-tailed, denoted by $\overline{F} \in \mathscr{L}$, if

$$\overline{F}(x+y) \sim \overline{F}(x), \quad -\infty < y < \infty.$$

It is well-known that

$$\mathscr{R} \subset \mathrm{ERV} \subset \mathscr{C} \subset \mathscr{L} \cap \mathscr{D}.$$

For the sums of pQAI and pSQAI rv's with heavy tails belonging to the class $\mathscr{L} \cap \mathscr{D}$, we have the following existing results, in which assertions (i) and (ii) are restatements of Theorem 3.1 of Chen and Yuen (2009) and Theorem 3.1 of Geluk and Tang (2009), respectively.

Proposition 1.1. *Let $X_1, \ldots, X_n$ be n real-valued rv's with survival functions $\overline{F}_1, \ldots, \overline{F}_n$. Then,*

$$\mathbb{P}\left(\sum_{i=1}^{n} X_i > x\right) \sim \sum_{i=1}^{n} \overline{F}_i(x) \tag{1.3}$$

holds if either (i) *$X_1, \ldots, X_n$ are pQAI and $\overline{F}_i \in \mathscr{C}$ for $1 \le i \le n$, or* (ii) *$X_1, \ldots, X_n$ are pSQAI and $\overline{F}_i \in \mathscr{L} \cap \mathscr{D}$ for $1 \le i \le n$.*

Enlightened by and starting from Proposition 1.1, in Section 2 we obtain some novel results regarding pQAI and pSQAI rv's within the class $\mathscr{L} \cap \mathscr{D}$. Our main concern lies in the asymptotics for constant and randomly weighted sums of pQAI and pSQAI rv's. Finally, in Section 3 we apply the obtained results to a kind of claim-dependent risk model and derive a precise asymptotic formula for the ultimate ruin probability.

2. Main results

Our first main theorem below indicates that, under the conditions of Proposition 1.1, relation (1.3) possesses an enhanced constant weighted version with the uniformity of the constant weights on any compact subset in $(0, \infty)$.

Theorem 2.1. *Under the conditions of either* (i) *or* (ii) *in Proposition 1.1, for every $0 < a \le b < \infty$, it holds uniformly for $(c_1, \ldots, c_n) \in [a, b]^n$ that*

$$\mathbb{P}\left(\sum_{i=1}^{n} c_i X_i > x\right) \sim \sum_{i=1}^{n} \overline{F}_i(x/c_i). \tag{2.1}$$