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On stationarity and β-mixing property of certain nonlinear GARCH(p,q) models ☆

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Abstract

Certain types of nonlinear GARCH (p,q) model which allows a signed volatility are considered. Sufficient conditions for strict stationarity and β-mixing with exponential decay rates are provided.

Keywords: GARCH model; Stationarity; Moments; β-Mixing

1. Introduction

Since Engle's (1982) and Bollerslev's (1986) seminal papers, ARCH-family models have been widely employed in the econometric and finance literature to model volatility. In classical GARCH model, conditional variance is a linear function of squared past disturbances and past observations. However, many data show that this linearity is not adequate and the conditional variances are asymmetric conditional on previous returns. In order to accommodate a variety of asymmetric volatilities, various GARCH-type models have been developed and successfully applied in finance and macroeconomics (see, e.g., Nelson, 1990; Ding et al., 1993; Zakoian, 1994; Duan, 1997; He and Teräsvirta, 1999 and references therein).

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Strict stationarity, ergodicity, existence of moments and mixing properties are fundamental for statistical inference and there is an extensive literature on these subjects. Bougerol and Picard (1992), Chen and An (1998) consider the linear GARCH model and investigate necessary and sufficient conditions for strict stationarity and existence of higher order moments. Properties of various GARCH(1,1) processes are studied by He and Teräsvirta (1999), Carrasco and Chen (2002), and others. An asymmetric power GARCH model is proposed by Ding et al. (1993) and results on moments and strict stationarity can be found in Ling and McAleer (2002). Duan (1997) proposes the augmented GARCH process and shows that the negative Lyapunov exponent is sufficient to ensure strict stationarity of the process. Carrasco and Chen (2002) study the mixing and moment properties of various GARCH models. Especially the GARCH(1,1)model is studied extensively.

In this paper, we consider a type of nonlinear GARCH(p, q) model defined as follows:

$$\varepsilon_t = \Lambda(h_t)\eta_t, \tag{1.1}$$

$$\begin{aligned} h_t = &\sum_{i=1}^{p} c_i(\eta_{t-1}, \ldots, \eta_{t-r_1}, h_{t-1}, \ldots, h_{t-s_1}) f(\eta_{t-i}) h_{t-i} \\ &+ \sum_{i=1}^{q} d_i(\eta_{t-1}, \ldots, \eta_{t-r_2}, h_{t-1}, \ldots, h_{t-s_2}) h_{t-i} \\ &+ g(\eta_{t-1}, \ldots, \eta_{t-r_3}, h_{t-1}, \ldots, h_{t-s_3}), \end{aligned} \tag{1.2}$$

where $\Lambda(\cdot)$ is a nonnegative continuous function, $p \geqslant 1$, $q, r, s \geqslant 0$, $r = \max(r_1, r_2, r_3), s = \max(s_1, s_2, s_3)$ are real constants, and c_i, d_i, f and g are measurable functions defined on $R^{r_1+s_1}$, $R^{r_2+s_2}$, R and $R^{r_3+s_3}$, respectively. Let $\{\eta_t\}$ be a sequence of independent and identically distributed (i.i.d.) random variables with mean zero and unit variance.

Note that the model of (1.1) and (1.2) includes broad classes of GARCH-type models, for example, a linear GARCH model of Bollerslev (1986), exponential GARCH model of Nelson (1991), GJR–GARCH model of Glosten et al. (1993), threshold GARCH model of Zakoian (1994), asymmetric power GARCH model of Ding et al. (1993), and augmented GARCH model suggested by Duan (1997), etc.

Our aim is to provide simple sufficient conditions under which the process ε_t generated by (1.1) and (1.2) is strictly stationary and/or β-mixing with exponential decay rates. For the case that model (1.1)–(1.2) can be represented as a generalized polynomial random coefficient autoregressive (RCA) model, we obtain conditions for the generalized RCA model to be geometrically ergodic and β-mixing, and then the desired results, stationarity and β-mixing for the process ε_t are derived. General GARCH-type models of (1.1) and (1.2) are also considered and sufficient conditions for strict stationarity are obtained.

For terminologies and relevant results in Markov chain theory, we refer to Meyn and Tweedie (1993).

2. Main result

We first consider a nonlinear GARCH(p, q) model, which is a simple case of (1.1) and (1.2), defined by

$$\varepsilon_t = \Lambda(h_t)\eta_t, \tag{2.1}$$