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Projection properties of some orthogonal arrays

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Abstract

In factor screening experiments, one generally starts with a large pool of potentially important factors. However, often only a few of these are really active. Under this assumption of effect sparsity, while choosing a design for factor screening, it is important to consider projections of the design on to smaller subsets of factors and examine whether the projected designs allow estimability of some interactions along with the main effects. While the projectivity properties of symmetric 2-level and a few 3-level fractional factorial designs represented by orthogonal arrays have been studied in the literature, similar studies in respect of asymmetric or, mixed level factorials seems to be lacking. In this paper, we initiate work in this direction by providing designs with good projectivity properties for asymmetric factorials of the type $t \times 2^m$ based on orthogonal arrays. We also note that the results of Cheng (1995) regarding the projectivity of symmetric two-symbol orthogonal arrays do not necessarily extend to arrays with more than two symbols.

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1. Introduction

In the initial stage of experimentation, one generally considers a large number of factors that might be potentially important. Among these, often only a few have large effects or, are active.

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Under this assumption of effect sparsity, studying the properties of projections of the design on to small subsets of factors becomes important as, the projected designs might allow the estimability of certain interactions among the projected factors, apart from that of the main effects. According to Box and Tyssedal (1996), a fractional factorial design is said to have projectivity p if in every subset of p factors, a complete factorial, with possibly some repeated runs is produced. Clearly, if a design has projectivity p and the number of active factors is at most p, the projection of the design on to the active factors allows the estimability of all factorial effects involving the active factors. The property of projectivity can be viewed as an extension of the strength of an orthogonal array. An orthogonal array, $\mathrm{OA}(N, n, m_1 \times \cdots \times m_n, g)$ of strength g, $2 \leqslant g < n$ is an $N \times n$ matrix, having $m_i \geqslant 2$ distinct symbols in the ith column, $i = 1, \ldots, n$, such that in any $N \times g$ submatrix, all possible combinations of the symbols occur equally often as a row. When $m_1 = \cdots = m_n = m$, the orthogonal array is called symmetric and is denoted by $\mathrm{OA}(N, n, m, g)$; otherwise, the array is called asymmetric. An $\mathrm{OA}(N, n, m_1 \times \cdots \times m_n, g)$ represents an N-run fractional factorial design for an asymmetric or mixed level $m_1 \times \cdots \times m_n$ experiment, with symbols representing the levels of the factors, columns corresponding to factors and rows representing the runs or, treatment combinations. Similarly, a symmetric orthogonal array $\mathrm{OA}(N, n, m, g)$ represents an N-run fractional factorial design for a symmetric m^n experiment. A fractional factorial plan represented by such an orthogonal array obviously has projectivity g.

In the case of symmetric factorials, an important class of symmetric orthogonal arrays give rise to the so-called regular fractional factorial designs. It is well known that a regular fractional factorial design of resolution R is an orthogonal array of strength $R-1$. Such a regular design has projectivity $R-1$ but cannot have projectivity greater than $R-1$. However, it is possible for a non-regular fractional factorial design represented by an orthogonal array of strength g to have projectivity greater than g. This fact was first observed by Lin and Draper (1992) and Box and Bisgaard (1993), who found that certain Plackett–Burman plans (Plackett and Burman, 1946) for 2-level symmetric factorials have projectivity three, even though it is known that such plans are represented by orthogonal arrays of strength two. Cheng (1995) proved that as long as N is not a multiple of 8, a fractional factorial design represented by an $\mathrm{OA}(N, n, 2, 2)$ has projectivity three. This result of Cheng (1995) extends the patterns observed by Lin and Draper (1992) and Box and Bisgaard (1993) on small Plackett–Burman designs through computer searches. The result of Box and Tyssedal (1996) is a special case of the result of Cheng (1995). Cheng (1995) further proved that as long as N is not a multiple of 16, a fractional factorial design represented by an $\mathrm{OA}(N, n, 2, 3)$ of strength three has projectivity four. However, these results of Cheng do not necessarily extend for (symmetric) orthogonal arrays with more than two symbols, as demonstrated by a counter-example in the next section.

Some non-regular fractions also exhibit a hidden projection property. A fractional factorial plan is said to have hidden projection property of order p if it allows the estimability of the main effects and all or, some two-factor interactions when projected on to any subset of p factors. For example, the 12-run Plackett–Burman plan has projectivity three, but when projected on to any four factors, has the property that all four main effects and two-factor interactions among the four are estimable, when higher-order effects are assumed negligible; see Lin and Draper (1992) and Wang and Wu (1995).