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Elliptical triangular arrays in the max-domain of attraction of Hüsler–Reiss distribution

Enkelejd Hashorva[a,b,*]

[a]Department of Statistics, University of Bern, Sidlerstrasse 5, CH-3012, Bern, Switzerland
[b]Actuarial Department, Allianz Suisse Insurance Company, Laupenstrasse 27, CH-3001 Bern, Switzerland

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Abstract

Let $(U_{ni}, V_{ni}),\ 1 \leqslant i \leqslant n$ be a triangular array of independent bivariate elliptical random vectors. Hüsler and Reiss (1989. Statist. Probab. Lett. 7, 283–286) show that for the particular case that the array is Gaussian, the maxima of this array is in the max-domain of attraction of Hüsler–Reiss distribution function, provided that an asymptotic condition holds for the correlation $\mathbf{corr}(U_{n1}, V_{n1})$. In this paper we obtain a similar result for the more general case of elliptical triangular arrays.

Keywords: Maxima of triangular arrays; Elliptical distribution; Hüsler–Reiss max-stable distribution; Weak convergence

1. Introduction

Consider $\{(E_{n1}, E_{n2}), n \geqslant 1\}$ a bivariate sequence of independent standard Gaussian random vectors with distribution function Φ_n. Let $\rho_n := \mathbf{corr}(E_{n1}, E_{n2}) \in (-1, 1)$ denote the correlation between E_{n1} and E_{n2}. Sibuya (1960) shows that if ρ_n does not depend on n then

$$\lim_{n\to\infty} \sup_{(x,y)\in\mathbb{R}^2} \left| \Phi_n^n(a(n)x + b(n), a(n)y + b(n)) - \exp(-\exp(-x) - \exp(-y)) \right| = 0, \tag{1}$$

*Corresponding address. Department of Statistics, University of Bern, Sidlerstrasse 5, CH-3012, Bern, Switzerland. Tel.: +41 31 384 5528; fax: +41 31 384 4572.

E-mail addresses: Enkelejd.Hashorva@Allianz-Suisse.ch, enkelejd@stat.unibe.ch.

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with

$$a(n) = \frac{1}{\sqrt{2 \ln n}}, \quad b(n) = \sqrt{2 \ln n} - \frac{1}{2\sqrt{2 \ln n}} \ln(4\pi \ln n).$$

Let further $(U_{ni}, V_{ni}), n > 1, 1 \leqslant i \leqslant n$ be a triangular array of independent bivariate standard Gaussian random vectors with common distribution function Φ_n and put $M_{n1} := \max_{1 \leqslant i \leqslant n} U_{ni}, M_{n2} := \max_{1 \leqslant i \leqslant n} V_{ni}$. Then (1) implies the convergence in distribution

$$([M_{n1} - b(n)]/a(n), [M_{n2} - b(n)]/a(n)]) \stackrel{d}{\to} (\mathscr{M}_1, \mathscr{M}_2), \quad n \to \infty,$$

with $\mathscr{M}_1$, $\mathscr{M}_2$ two independent random variables with distribution function $\Lambda(x) = \exp(-\exp(-x)), x \in \mathbb{R}$, (the well-known standard Gumbel distribution). Hence the maxima of such Gaussian triangular arrays has asymptotic independent components. This result seems at the first sight very surprising recalling that the correlation $\rho_n \neq 0$. So despite the fact that Φ_n is not a product distribution, the maxima has asymptotically independent components.

Indeed, there are many examples of multivariate distribution functions that have the same property. In general for such distributions, there is no direct link between correlation ρ_n and asymptotic independence. In the Gaussian case, however, we have the following equality in distribution

$$(E_{n1}, E_{n2}) \stackrel{d}{=} (E_*, \rho_n E_* + \sqrt{1 - \rho_n^2} E_{**}) \tag{2}$$

with E_*, E_{**} two independent standard Gaussian random variables. Letting $\rho_n \to 1$, thus imposing the random vector (E_{n1}, E_{n2}) to behave asymptotically $(n \to \infty)$ like (E_*, E_*) suggests that the maxima of the triangular array above may have eventually dependent components. Indeed this can be achieved by imposing a certain rate of convergence to 0 for $1 - \rho_n$. More specifically, Theorem 2 of Hüsler and Reiss (1989) (see also Reiss, 1989) shows that if ρ_n is such that

$$\lim_{n \to \infty} (1 - \rho_n) \ln n = \lambda^2 \in [0, \infty] \tag{3}$$

holds, then the convergence in distribution

$$([M_{n1} - b(n)]/a(n), [M_{n2} - b(n)]/a(n)) \stackrel{d}{\to} (\mathscr{M}_1^*, \mathscr{M}_2^*) \tag{4}$$

holds, where the limiting random vector $(\mathscr{M}_1^*, \mathscr{M}_2^*)$ has distribution function given by

$$H_\lambda(x, y) = \exp\left(-\Phi\left(\lambda + \frac{x - y}{2\lambda}\right) \exp(-y) - \Phi\left(\lambda + \frac{y - x}{2\lambda}\right) \exp(-x)\right), \quad x, y \in \mathbb{R},$$

with Φ the univariate standard Gaussian distribution. For $\lambda = 0$ and ∞ the asymptotic complete dependence and independence of the components holds respectively in the limit i.e.

$$H_0(x, y) = \min(\Lambda(x), \Lambda(y))$$

and

$$H_\infty(x, y) = \Lambda(x)\Lambda(y),$$

with $\Lambda(x) = \exp(-\exp(-x)), x \in \mathbb{R}$ the standard Gumbel distribution function. Hüsler and Reiss (1989) show further that the distribution function H_λ is max-stable.