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Iterated integrals with respect to Bessel processes

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Abstract

Let $X=(X_t)_{t\geqslant 0}$ be the square of a δ ($\geqslant 0$)-dimensional Bessel process starting at zero. Define iterated stochastic integrals $I_n(t,\delta)$, $t\geqslant 0$ inductively by

$$I_n(t,\delta)=\int_0^t I_{n-1}(s,\delta)\,\mathrm{d}X_s$$

with $I_0(t,\delta)=1$ and $I_1(t,\delta)=X_t$. Then the inequalities

$$c_{n,p,\delta}\|\tau^n\|_p\leqslant\left\|\sup_{0\leqslant t\leqslant\tau}|I_n(t,\delta)|\right\|_p\leqslant C_{n,p,\delta}\|\tau^n\|_p$$

and

$$c_{n,p,\delta}\|G_\delta(\tau)^n\|_p\leqslant\left\|\sup_{0\leqslant t\leqslant\tau}|I_n(t,\delta)|/(1+t)^n\right\|_p\leqslant C_{n,p,\delta}\|G_\delta(\tau)^n\|_p$$

are proved to hold for all $0<p<\infty$ and all stopping times τ, where c,C are some positive constants depending only on the subscripts, and $G_\delta(t)=\log(1+\delta\log(1+t))$.

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1. Introduction and results

Throughout this paper, we shall work with a filtered complete probability space $(\Omega, \mathscr{F}, (\mathscr{F}_t), P)$ satisfying the usual conditions, and let $B = (B_t)_{t\geqslant 0}$ be a standard Brownian motion starting at zero. For any continuous process X we denote $X_t^* = \sup_{0\leqslant s\leqslant t}|X_s|$ and $X^* = X_\infty^*$. Let C stand for a positive constant depending only on the subscripts and its value may be different in different appearance, and furthermore, this assumption is also adaptable to c.

Carlen and Krée (1991) considered iterated stochastic integrals

$$I_n(B) = (I_n(t, B), \mathscr{F}_t) \quad (n\geqslant 0),$$

defined inductively by

$$I_n(t, B) = \int_0^t I_{n-1}(s, B)\,\mathrm{d}B_s$$

with $I_0(t, B) = 1$ and $I_1(t, B) = B_t$. They established L^p-estimates on $I_n(B)$ as follows:

$$c_{n,p}\|\tau^{n/2}\|_p \leqslant \|I_n(\tau, B)\|_p \leqslant C_{n,p}\|\tau^{n/2}\|_p$$

for all stopping times τ, where the right side holds for $p\geqslant 1$ and the left side for $p>1$. In this paper, we consider the iterated stochastic integrals with respect to the square of a δ ($\geqslant 0$)-dimensional Bessel process starting at zero. Our aims are to prove the following theorems.

Theorem 1.1. *Let $X = (X_t)_{t\geqslant 0}$ be the square of a δ ($\geqslant 0$)-dimensional Bessel process starting at zero. Define iterated stochastic integrals $I_n(t, \delta)$, $t\geqslant 0$ inductively by*

$$I_n(t, \delta) = \int_0^t I_{n-1}(s, \delta)\,\mathrm{d}X_s \tag{1.1}$$

with $I_0(t, \delta) = 1$ and $I_1(t, \delta) = X_t$. Then the inequalities

$$c_{n,p,\delta}\|\tau^n\|_p \leqslant \left\|\sup_{0\leqslant t\leqslant\tau}|I_n(t, \delta)|\right\|_p \leqslant C_{n,p,\delta}\|\tau^n\|_p \tag{1.2}$$

hold for all $0<p<\infty$ and all stopping times τ.

Theorem 1.2. *Let $G_\delta(t) = \log(1 + \delta\log(1 + t))$, $\delta>0$. Under the condition of Theorem 1.1 we have*

$$c_{n,p,\delta}\|(G_\delta(\tau))^n\|_p \leqslant \left\|\sup_{0\leqslant t\leqslant\tau}\frac{|I_n(t, \delta)|}{(1 + t)^n}\right\|_p \leqslant C_{n,p,\delta}\|(G_\delta(\tau))^n\|_p \tag{1.3}$$

for all $0<p<\infty$ and all stopping times τ.

2. Bessel processes with nonnegative dimension

In this section, we first recall that the square representation of the dimension $\delta(>0)$-Bessel process.

Consider the stochastic differential equation

$$\mathrm{d}X_t = \delta\,\mathrm{d}t + 2\sqrt{|X_t|}\,\mathrm{d}B_t, \quad X_0 = x\geqslant 0, \tag{2.1}$$