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Type G and spherical distributions on $\mathbb{R}^d$

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Abstract

A class of multivariate distributions obtained by Gaussian randomizations of jumps of a Lévy process is studied. Specifically, exact convenient representations of type G distributions, given that they are of spherical type, are demonstrated. The methodology reveals new ways in extracting families of distributions that may help in understanding various applications that arise in finance. Applications from explicit distributions are also confirmed.

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1. Introduction

The standard models for returns in portfolio allocation (Merton, 1971) and option pricing (Black and Scholes, 1973) assume that continuously compounded returns are normally distributed. The central limit theorem is often invoked as the primary motivation for this assumption. Unfortunately, it is well known that the normal assumption of the return distribution

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is violated in both the time series data and in option prices (see e.g., Barndorff-Nielsen, 1998; Barndorff-Nielsen and Shephard, 2001; Heston, 1993; Madan et al., 1998). Thus, the use of general Lévy processes instead of the traditional Wiener process make it possible to incorporate processes with jumps and infinite variance. The type G family of processes is a subclass of Lévy processes, which allows retaining some of the Gaussian properties of the Wiener process and yet is rich enough to include processes with jumps and infinite variance.

Marcus (1987) initially introduced the concept of type G random variables and processes. Since then, the type G family has caught the attention of many investigators including Rosinski (1991), Maejima and Rosinski (2002, 2001), Maejima and Samorodnitsky (1999) and Barndorff-Nielsen and Pérez-Abreu (2003).

An $\mathbb{R}^d$-valued random vector is said to be of type G if it can be represented as follows:

$$\mathbf{X} := V^{1/2}\mathbf{G}, \tag{1.1}$$

where $\mathbf{G}$ is a mean zero Gaussian random vector in $\mathbb{R}^d$, with positive definite variance–covariance matrix Σ. We let the matrix V belong to the closed cone of $d \times d$ non-negative symmetric definite and infinite divisible random matrices. Write $V^{1/2}$ to denote its square root of V.

Random vectors of the form (1.1) were introduced in Barndorff-Nielsen and Pérez-Abreu (2003) and Maejima and Rosinski (2002). Maejima and Rosinski (2002) noticed that type G processes could be related to Lévy processes composed of random matrices as subordinator-type. Specifically, we let $\{\mathbf{G}(t) : t \geqslant 0\}$ be a Lévy Gaussian process with variance–covariance Σ. Further, we construct the matrix process $V(t)$ from a vector process $\{\mathbf{V}_0(t) \in \mathbb{R}^d : t \geqslant 0\}$. It is assumed that $\{\mathbf{V}_0(t) \in \mathbb{R}^d : t \geqslant 0\}$ is a Lévy process with no Gaussian component whose quadratic variation satisfies

$$[\mathbf{V}_0, \mathbf{V}_0](t) := V(t). \tag{1.2}$$

Thus, the process $\{\mathbf{X}(t) \in \mathbb{R}^d : t \geqslant 0\}$ is obtained from the Lévy process $\{\mathbf{V}_0(t) \in \mathbb{R}^d : t \geqslant 0\}$ as follows:

$$\mathbf{X} = \mathbf{G} \circ V \text{ i.e., } \mathbf{X}_t = \mathbf{G}_{[\mathbf{V}_0, \mathbf{V}_0](t)}, \quad t \geqslant 0. \tag{1.3}$$

Clearly, the process $\mathbf{X}$ is constructed from $\mathbf{G}$ by an independent random process matrix at time t.

For the purpose of analyzing the process (1.3), a more convenient formulation is proposed. Specifically, we let v_0 be a symmetric Lévy measure of the vector $\mathbf{V}_0 := \mathbf{V}_0(1) \in \mathbb{R}^d$. We assume that $\mathbf{X}$ is of type G with no Gaussian component. We also assume that $v_0(\mathbb{R}^d) < \infty$. Following Maejima and Rosinski (2002), we further define $\{\mathbf{Y}_j \in \mathbb{R}^d : j \in \mathbf{N}\}$ to be an independent and identically distributed (*i.i.d.*) sequence of random vectors with common distribution $v_0/v_0(\mathbb{R}^d)$, and let $\{N(t) \in \mathbf{N} : t \geqslant 0\}$ be a Poisson process with intensity $v_0(\mathbb{R}^d)$. Let $\{Z_j \in \mathbb{R} : j \in \mathbf{N}\}$ be an *i.i.d.* sequence of standard random variables and Σ be a positive definite symmetric matrix. Assume that $\{\mathbf{Y}_j \in \mathbb{R}^d : j \in \mathbf{N}\}$, $\{N(t) \in \mathbf{N} : t \geqslant 0\}$ and $\{Z_j \in \mathbb{R} : j \in \mathbf{N}\}$ are independent of each other. Replacing the random vector $\mathbf{Y}_j$, $j \in \mathbf{N}$, in Maejima and Rosinski (2002) by $\Sigma^{1/2}\mathbf{Y}_j$, $j \in \mathbf{N}$, the random vector $\mathbf{X}$ now has the following representation

$$\mathbf{X} := \sum_{j=1}^{N(1)} \Sigma^{1/2}\mathbf{Y}_j Z_j. \tag{1.4}$$