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Test for the equality of autocorrelation coefficients for two populations in multivariate data when the errors are autocorrelated

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Abstract

We derive a likelihood ratio test for the equality of two autocorrelation coefficients based on two independent multinormal samples. The critical value and the power analysis of the test are obtained through simulation.

Keywords: Likelihood ratio test; Autocorrelation coefficients

1. Introduction

The autocorrelation coefficient ρ is frequently used to measure the autocorrelation in a time-series model. Weather patterns throughout the year changes month by month and there is autocorrelation of the weather patterns from one month to the next month. Similarly, behavior of stock-market pattern from day to day have autocorrelation effect.

Statistical inference concerning ρ for a single sample problem has been studied by Durbin and Watson (1950, 1951, 1971). Some discussions are also given in Morrison (1983). Cochrane and Orcutt (1949) has a discussion about estimating the regression parameters when the errors are autocorrelated. Surprisingly, the extension of inference problem concerning ρ to two-sample as well as multisample problems has received very little attention.

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In this paper, we consider the problem of testing the equality of two autocorrelation coefficients based on two independent multinormal samples. It could be of interest to see whether the autocorrelation of weather patterns throughout the year in United States differs from the same in Asia and therefore, we need to develop test for testing the equality of autocorrelation coefficients.

In Section 2, we derive the likelihood ratio test for the equality of autocorrelation coefficients . The critical value and the power analysis of the test are obtained through simulation and are discussed in Section 3.

2. Test of $H_0 : \rho_1 = \rho_2$ versus $H_1 : \rho_1 \neq \rho_2$

2.1. Likelihood ratio test

The model for the multivariate data with autocorrelated error is as follows: $\underset{\sim}{x} = \underset{\sim}{\mu} + \underset{\sim}{\varepsilon}$, where

$\underset{\sim}{x} = (x_1 \ldots x_p)'$ is a $p \times 1$ vector of observations,

$\underset{\sim}{\mu} = (\mu_1 \ldots \mu_p)'$ is a $p \times 1$ vector of unknown means,

$\underset{\sim}{\varepsilon} = (\varepsilon_1 \ldots \varepsilon_p)'$ is a $p \times 1$ vector of random errors.

It is assumed that $\underset{\sim}{\varepsilon} \sim N_p(\underset{\sim}{0}, \Sigma)$, where

$$\Sigma = \sigma^2 \begin{bmatrix} 1 & \rho & \rho^2 & \ldots & \rho^{p-1} \\ \rho & 1 & \rho & \ldots & \rho^{p-2} \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ \rho^{p-1} & \rho^{p-2} & \ldots & \ldots & 1 \end{bmatrix}, \tag{2.1}$$

and N_p denotes p-variate normal distribution. The structure of the covariance matrix in (2.1) means that the errors are autocorrelated. The autocorrelatedness of the error is quite common in real practice. In fact, it can be tested from the data whether the error covariance structure is of (2.1) or not.

In expression (2.1), σ^2 represents the variance of each error component and ρ is called the autocorrelation coefficient.

Let $\underset{\sim}{x_1}, \underset{\sim}{x_2}, \ldots, \underset{\sim}{x_n}$ be $p \times 1$ vector of n observations independently and identically distributed as $N_p(\underset{\sim}{\mu}, \Sigma)$, where Σ is given by (2.1). We can make the following transformation: $\underset{\sim}{y_i} = T\underset{\sim}{x_i}, i = 1, 2, \ldots, n$, where

$$T = \begin{bmatrix} \sqrt{1-\rho^2} & 0 & 0 & \ldots & 0 \\ -\rho & 1 & 0 & \ldots & 0 \\ 0 & -\rho & 1 & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & 0 & -\rho & 1 \end{bmatrix}. \tag{2.2}$$