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Excited Mob

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Abstract

We study one dimensional excited random walks (ERW) on iterated leftover environments. We prove a 0–1 law for directional transience and a law of large numbers for such environments under mild assumptions. We provide exact criteria for transience and positive speed of the walk in terms of the expected drift per site under stronger assumptions. This allows us to construct examples of stationary and ergodic environments on which ERW has positive speed that do not follow by trivial comparison to i.i.d. environments. A central ingredient is the introduction of the "Excited Mob" of k walkers on the same cookie environment.

MSC: 60K35; 60K37; 60J80

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1. Introduction

1.1. Basic model and notations

Excited random walk on $\mathbb{Z}^d,\ d \geq 1$, was introduced by Itai Benjamini and David B. Wilson in 2003 [4]. The model in dimension $d = 1$ was generalized by Martin P.W. Zerner [20]. It was studied extensively in recent years by numerous authors, and an almost up to date account may be found in the recent survey of Kosygina and Zerner [13].

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The model is defined as follows. Let $\Omega = [0,1]^{\mathbb{Z}\times\mathbb{N}}$ and endow this space with the Borel σ-algebra generated by the Tychonoff product topology, where the space $[0,1]$ has the standard real topology. We call Ω the space of cookie environments, where each $\omega \in \Omega$ is a cookie environment. $\omega(x,n) \in [0,1]$ is called the nth cookie in the location x.

Given a cookie environment ω and an initial position $x \in \mathbb{Z}$ the excited random walk $X = (X_n)_{n\geq 0}$ driven by ω is given by: $\mathbb{P}_{\omega,x}(X_0 = x) = 1$,

$$\mathbb{P}_{\omega,x}(X_n = X_{n-1} + 1 \mid \mathcal{F}_{n-1}) = \omega(X_{n-1}, \#\{k \leq n-1 : X_k = X_{n-1}\}),$$
$$\mathbb{P}_{\omega,x}(X_n = X_{n-1} - 1 \mid \mathcal{F}_{n-1}) = 1 - \mathbb{P}_{\omega,x}(X_n = X_{n-1} + 1 \mid \mathcal{F}_{n-1}).$$

Here $\mathcal{F}_n = \sigma(\{X_0, X_1, \ldots, X_n\})$ is the σ-algebra generated by the first n positions of the walk, $n \geq 0$. The probability measure $\mathbb{P}_{\omega,x}$ is called the *quenched* measure on the excited random walks started from x. Given a probability measure P on the space Ω of cookie environments, with a corresponding expectation operator E, we define the *annealed* (also called *averaged*) measure $\mathbb{P}_x$ to be the P-average of the quenched measure:

$$\mathbb{P}_x(\cdot) = E[\mathbb{P}_{\omega,x}(\cdot)].$$

The following two assumptions on the measure P on cookie environments are standard. We adopt the notations of [13]. In particular $\mathbb{N} := \{1, 2, \ldots\}$ is the set of positive integers, while $\mathbb{N}_0 := \mathbb{N} \cup \{0\}$ is the set of non negative integers.

$$\text{The family } (\omega(x,\cdot))_{x\in\mathbb{Z}} \text{ of cookie stacks is i.i.d. under } P \tag{IID}$$

and

$$\begin{array}{l}\text{The family } (\omega(x,\cdot))_{x\in\mathbb{Z}} \text{ is stationary and ergodic}^{1} \text{ under } P\\ \text{with respect to the shift on } \mathbb{Z}.\end{array} \tag{SE}$$

Define also the following properties of a cookie environment ω: ellipticity, non-degeneracy, and positivity.

$$\omega(x,n) \in (0,1) \quad \text{for all } x \in \mathbb{Z} \text{ and } n \in \mathbb{N}. \tag{ELL}$$

$$\sum_{i=1}^{\infty} \omega(x,i) = \infty \quad \text{and} \quad \sum_{i=1}^{\infty} (1 - \omega(x,i)) = \infty \quad \text{for all } x \in Z. \tag{ND}$$

$$\omega(x,n) \geq \frac{1}{2} \quad \text{for all } x \in \mathbb{Z} \text{ and } n \in \mathbb{N}. \tag{POS}$$

Say that a probability measure P on Ω satisfies (ELL), (ND) or (POS), respectively if P-a.s. ω satisfies it. The Non-degeneracy condition (ND) implies that for almost every environment ω, the walk is either transient or a.s. visits all vertices infinitely often. (Without it other behaviors, such as being stuck in a finite interval, are possible).

Last, we define two additional properties on P: boundedness and weak ellipticity.

$$\begin{array}{l}\text{There is some deterministic } M \text{ such that } P\text{-a.s.}\\ \omega(x,n) = \dfrac{1}{2} \quad \text{for all } x \in \mathbb{Z} \text{ and } n > M.\end{array} \tag{BD}$$

$$\text{For all } x \in \mathbb{Z}\ P(\omega(x,n) > 0\ \forall n \in \mathbb{N}) > 0 \quad \text{and} \quad P(\omega(x,n) < 1\ \forall n \in \mathbb{N}) > 0. \tag{WEL}$$

[1] I.e. every Borel measurable $A \subset \Omega$ which is invariant under left or right shifts on $\mathbb{Z}$ satisfies $P(A) \in \{0,1\}$.