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Pinning model with heavy tailed disorder

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Abstract

We study the pinning model, which describes the behavior of a Markov chain interacting with a distinguished state. The interaction depends on an external source of randomness, called disorder. Inspired by Auffinger and Louidor (2011) and Hambly and Martin (2007), we consider the case when the disorder is heavy-tailed, while the return times of the Markov chain are stretched-exponential. We prove that the set of times at which the Markov chain visits the distinguished state, suitably rescaled, has a limit in distribution. Moreover there exists a random threshold below which this limit is trivial. Finally we complete a result of Auffinger and Louidor (2011) on the directed polymer in a random environment.

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Keywords: Pinning model; Directed polymers; Heavy tails; Localization

1. Set-up and results

The pinning model can be defined as a random perturbation of a random walk or, more generally, of a Markov chain called S. In this model we modify the law of the Markov chain by weighing randomly the probability of a given trajectory up to time N. Each time S touches a distinguished state, called 0, before N, say at time n, we give a reward or a penalty to this contact by assigning an exponential weight $\exp(\beta\omega_n - h)$, where $\beta \in \mathbb{R}_+ := (0, \infty)$, $h \in \mathbb{R}$ and $(\omega = (\omega_n)_{n\in\mathbb{N}}, \mathbb{P})$ is an independent random sequence called disorder. The precise definition of the model is given below.

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In this model we perturb S only when it takes value 0, therefore it is convenient to work with its zero level set. For this purpose we consider a renewal process $(\tau = (\tau_n)_{n\in\mathbb{N}}, \mathrm{P})$, that is an $\mathbb{N}_0$-valued random process such that $\tau_0 = 0$ and $(\tau_j - \tau_{j-1})_{j\in\mathbb{N}}$ is an i.i.d. sequence. This type of random process can be thought of as a random subset of $\mathbb{N}_0$, in particular if $S_0 = 0$, then by setting $\tau_0 = 0$ and $\tau_j = \inf\{k > \tau_{j-1} : S_k = 0\}$, for $j > 0$, we recover the zero level set of the Markov chain S. From this point of view the notation $\{n \in \tau\}$ means that there exists $j \in \mathbb{N}$ such that $\tau_j = n$. We refer to [1,9] for more details about the theory of the renewal processes.

In the literature, e.g. [7,10,9], typically the law of τ_1, the inter-arrival law of the renewal process, has a polynomial tail and the disorder has finite exponential moments. In our paper we study the case in which the disorder has polynomial tails, in analogy with the articles [2, 11]. To get interesting results we work with a renewal process where the law of τ_1 is stretched-exponential (cf. Assumption 1.2). Possible generalizations will be discussed in Section 7.

1.1. The pinning model

In this paper we want to understand the behavior of $\tau/N \cap [0, 1] = \{\tau_j/N : \tau_j \leq N\}$, the rescaled renewal process up to time N,when N gets large.

We denote by P_N the law of $\tau/N \cap [0, 1]$, which turns out to be a probability measure on the space of all subsets of $\{0, 1/N, \dots, 1\}$. On this space, for $\beta, h \in \mathbb{R}$ we define the *pinning model* $\mathrm{P}^{\omega}_{\beta,h,N}$ as a probability measure defined by the following Radon–Nikodym derivative

$$\frac{\mathrm{dP}^{\omega}_{\beta,h,N}}{\mathrm{dP}_N}(I) = \frac{1}{\mathrm{Z}^{\omega}_{\beta,h,N}} \exp\left(\sum_{n=1}^{N-1}(\beta\omega_n - h)\mathbb{1}(n/N \in I)\right)\mathbb{1}(1 \in I), \tag{1.1}$$

where $\mathrm{Z}^{\omega}_{\beta,h,N}$ is a normalization constant, called partition function, that makes $\mathrm{P}^{\omega}_{\beta,h,N}$ a probability. Let us stress that a realization of $\tau/N \cap [0, 1]$ has non-zero probability only if its last point is equal to 1. This is due to the presence of the term $\mathbb{1}(1 \in I)$ in (1.1). In such a way the pinning model is a *random* probability measure on the space X of all closed subsets of [0, 1] which contain both 0 and 1

$$\mathrm{X} = \{I \subset [0, 1] : I \text{ is closed and } 0, 1 \in I\} \tag{1.2}$$

with support given by $\mathrm{X}^{(N)}$, the set of all subsets of $\{0, 1/N, \dots, 1\}$ which contains both 0 and 1.

The pinning model $\mathrm{P}^{\omega}_{\beta,h,N}$ is a *random* probability measure, in the sense that it depends on a parameter ω, called disorder, which is a quenched realization of a random sequence. Therefore in the pinning model we have two (independent) sources of randomness: the renewal process (τ, P) and the disorder $(\omega, \mathbb{P})$. To complete the definition we thus need to specify our assumptions about the disorder and the renewal process.

Assumption 1.1. We assume that the disorder ω is an i.i.d. sequence of random variables whose tail is regularly varying with index $\alpha \in (0, 1)$, namely

$$\mathbb{P}(\omega_1 > t) \sim L_0(t)t^{-\alpha}, \quad t \to \infty, \tag{1.3}$$

where $\alpha \in (0, 1)$ and $L_0(\cdot)$ is a slowly varying function, cf. [5]. Moreover we assume that the law of ω_1 has no atom and it is supported in $(0, \infty)$, i.e. ω_1 is a positive random variable. The reference example to consider is given by the Pareto Distribution.

Assumption 1.2. Given a renewal process, we denote the law of its first point τ_1 by $K(n) := \mathrm{P}(\tau_1 = n)$, which characterizes completely the process. Throughout the paper we consider a