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Stochastic Processes and their Applications xx (xxxx) xxx–xxx

stochastic processes and their applications

www.elsevier.com/locate/spa

Robust no-free lunch with vanishing risk, a continuum of assets and proportional transaction costs

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Received 2 February 2013; received in revised form 3 February 2014; accepted 24 April 2014

Abstract

We propose a continuous time model for financial markets with proportional transaction costs and a continuum of risky assets. This is motivated by bond markets in which the continuum of assets corresponds to the continuum of possible maturities. Our framework is well adapted to the study of no-arbitrage properties and related hedging problems. In particular, we extend the Fundamental Theorem of Asset Pricing of Guasoni, Rásonyi and Lépinette (2012) which concentrates on the one dimensional case. Namely, we prove that the Robust No Free Lunch with Vanishing Risk assumption is equivalent to the existence of a Strictly Consistent Price System. Interestingly, the presence of transaction costs allows a natural definition of trading strategies and avoids all the technical and un-natural restrictions due to stochastic integration that appear in bond models without friction. We restrict to the case where exchange rates are continuous in time and leave the general càdlàg case for further studies.

MSC: 91B25; 60G44

Keywords: No-arbitrage; Transaction costs; Continuous time bond market

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http://dx.doi.org/10.1016/j.spa.2014.04.012
0304-4149/

1. Introduction

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The main contribution of this paper is to construct a continuous time model for financial markets with proportional transaction costs allowing for a continuum of risky assets. Such a model should have two important properties: 1. Financial strategies should be defined in a natural way; 2. It should allow one to retrieve the main results already established in the "finite dimensional price" case. Our model has both.

Frictionless models with a continuum of assets have already been proposed in the literature, cf. [2,8,14,26]. However, working with infinite dimensional objects leads to important technical difficulties when it comes to stochastic integration. This imposes non-natural restrictions on the set of admissible trading strategies, resulting in that even markets with a *unique equivalent martingale measure* are incomplete, in the sense that the set of attainable bounded claims is generically only dense in L^∞ and not closed. Other surprising pitfalls and counter-intuitive results were pointed out in [27].

Introducing transaction costs allows one to reduce these problems. The main reason is that it naturally leads to a definition of wealth processes which does not require stochastic integration. Once frictions are introduced, one comes up with a more realistic but also more natural and somehow simpler model.

In [4], the authors studied for the first time an infinite dimensional setting within the family of models with proportional transaction costs. They considered a countable number of assets in a discrete time framework, and imposed a version of the *efficient friction* condition, namely that the duals of the solvency cones have non-empty interior. Since perfectly adapted to discrete time models, they studied the No-Arbitrage of Second Kind (NA2) condition, first introduced in [21,22]. They showed that it implies the Fatou closure property of the set of super-hedgeable claims and noted that this closure property is in general lost if the efficient friction condition is replaced by a weaker condition, such as only requiring the solvency cones to be proper (as in finite dimensional settings).

In [4] also a dual equivalent characterization was given in terms of Many Strictly Consistent Price Systems (MSCPS condition), cf. [20,21]. These price systems are the counterpart of the martingale measures in frictionless markets, i.e. the building blocks of dual formulations for derivative pricing and portfolio management problems.

The main contribution of the present paper is to provide an extension of this model to a continuous time setting with a continuum of assets: the price process is, roughly speaking (for details see (2.1)–(2.3)), a continuous process on a time interval $[0, T]$ with values in the space $C([0, \infty])$ of continuous functions on $[0, \infty]$, the assets being indexed by the elements in $[0, \infty]$. A portfolio process is then a process of bounded variations, taking values in the space of Radon measures $M([0, \infty])$ on $[0, \infty]$, i.e. the dual of $C([0, \infty])$, when endowed with its sup norm. Taking into account the infinite dimension, we develop this into a Kabanov geometrical framework (cf. [20] for the finite dimensional case), with locally compact instantaneous solvency cones in $M([0, \infty])$ endowed with its weak* topology, their dual cones being viewed as subsets of $C([0, \infty])$.

Within this model, we study the No-Free Lunch with Vanishing Risk property, which is admitted to be the natural no-arbitrage condition in continuous time frictionless markets since the seminal paper of Delbaen and Schachermayer [10]. As [16], we consider a robust version (hereafter RNFLVR), *robust* being understood in the sense of [25], see also [17]: the no-arbitrage property should also hold for a model with slightly smaller transaction cost rates. It is now standard in the continuous time literature.

Please cite this article in press as: B. Bouchard, et al., Robust no-free lunch with vanishing risk, a continuum of assets and proportional transaction costs, Stochastic Processes and their Applications (2014), http://dx.doi.org/10.1016/j.spa.2014.04.012