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Factorial moments of point processes

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Abstract

We derive joint factorial moment identities for point processes with Papangelou intensities. Our proof simplifies previous combinatorial approaches to the computation of moments for point processes. We also obtain new explicit sufficient conditions for the distributional invariance of point processes with Papangelou intensities under random transformations.

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Keywords: Point process; Papangelou intensity; Factorial moment; Moment identity

1. Introduction

Consider the compound Poisson random variable

$$\beta_1 Z_{\alpha_1} + \cdots + \beta_p Z_{\alpha_p} \tag{1.1}$$

where $\beta_1, \ldots, \beta_p \in \mathbb{R}$ are constant parameters and $Z_{\alpha_1}, \ldots, Z_{\alpha_p}$ is a sequence of independent Poisson random variables with respective parameters $\alpha_1, \ldots, \alpha_p \in \mathbb{R}_+$. The Lévy–Khintchine

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formula

$$\mathbb{E}[e^{t(\beta_1 Z_{\alpha_1}+\cdots+\beta_p Z_{\alpha_p})}] = e^{\alpha_1(e^{\beta_1 t}-1)+\cdots+\alpha_p(e^{\beta_p t}-1)}$$

shows that the cumulant of order $k \geq 1$ of (1.1) is given by

$$\alpha_1 \beta_1^k + \cdots + \alpha_p \beta_p^k.$$

As a consequence, the moment of order $n \geq 1$ of (1.1) is given by the Faà di Bruno formula as

$$\mathbb{E}\left[\left(\sum_{i=1}^p \beta_i Z_{\alpha_i}\right)^n\right] = \sum_{m=1}^n \sum_{P_1\cup\cdots\cup P_m=\{1,\ldots,n\}} \sum_{i_1,\ldots,i_m=1}^p \beta_{i_1}^{|P_1|}\alpha_{i_1}\cdots\beta_{i_m}^{|P_m|}\alpha_{i_m}, \tag{1.2}$$

where the above sum runs over all partitions $P_1, \ldots, P_m$ of $\{1, \ldots, n\}$.

Such cumulant-type moment identities have been extended in [8] to Poisson stochastic integrals of random integrands through the use of the Skorohod integral on the Poisson space, cf. [6,7]. The construction of the Skorohod integral has been extended to point processes with Papangelou intensities in [9], and in [3], the moment identities of [8] have been extended to point processes with Papangelou intensities via a simpler combinatorial argument based on induction.

In this paper we deal with factorial moments, which are known to be easier to handle than standard moments, cf. for example, the direct relation between factorial moments and the correlation functions of point processes (2.3). See also [2] for the use of factorial moments to light-traffic approximations in queueing processes. In the case of random sets we obtain natural factorial moment identities by a direct induction argument, see Propositions 2.1 and 2.2. The moment identities of [3] can then be recovered from standard relations between factorial moments and classical moments.

On the other hand, our results allow us to derive new practicable sufficient conditions for the distributional invariance of point processes, with Papangelou intensities, cf. Condition (3.8) in Proposition 3.2. Similar results have been given in [3], however our conditions are different and are shown to be satisfied on typical examples that include transformations acting within the convex hull generated by the point process.

This paper is organized as follows. In Section 2, we derive factorial moment identities for random point measure of random sets in Propositions 2.1 and 2.2, and in Section 3 we apply those identities to point process transformations in Proposition 3.2. In Section 4, we show that the corresponding moment identities can be recovered by combinatorial arguments, cf. Proposition 4.2. In Section 5, we recover some recent results on the invariance of Poisson random measures under interacting transformations, with simplified proofs.

Notation and preliminaries on Papangelou intensities

Let X be a Polish space equipped with a σ-finite measure $\sigma(dx)$. Let Ω^X denote the space of configurations whose elements $\omega \in \Omega^X$ are identified with the Radon point measures $\omega = \sum_{x\in\omega} \epsilon_x$, where ϵ_x denotes the Dirac measure at $x \in X$. A point process is a probability measure P on Ω^X equipped with the σ-algebra $\mathcal{F}$ generated by the topology of vague convergence.

Point processes can be characterized by their Campbell measure C defined on $\mathcal{B}(X) \otimes \mathcal{F}$ by

$$C(A \times B) := \mathbb{E}\left[\int_X \mathbf{1}_A(x)\mathbf{1}_B(\omega \setminus \{x\})\, \omega(dx)\right], \quad A \in \mathcal{B}(X),\ B \in \mathcal{F}.$$