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Marc Arnaudon, Laurent Miclo

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A stochastic algorithm finding generalized means on compact manifolds

Marc Arnaudon[†] and Laurent Miclo[‡]

[†] Institut de Mathématiques de Bordeaux, UMR 5251
Université de Bordeaux and CNRS, France

[‡] Institut de Mathématiques de Toulouse, UMR 5219
Université de Toulouse and CNRS, France

Abstract

A stochastic algorithm is proposed, finding the set of generalized means associated to a probability measure ν on a compact Riemannian manifold and a continuous cost function κ on $M \times M$. Generalized means include p-means for $p \in (0, \infty)$, computed with any continuous distance function, not necessarily the Riemannian distance. They also include means for lengths computed from Finsler metrics, or for divergences.

The algorithm is fed sequentially with independent random variables $(Y_n)_{n\in\mathbb{N}}$ distributed according to ν and this is the only knowledge of ν required. It evolves like a Brownian motion between the times it jumps in direction of the Y_n. Its principle is based on simulated annealing and homogenization, so that temperature and approximations schemes must be tuned up. The proof relies on the investigation of the evolution of a time-inhomogeneous $\mathbb{L}^2$ functional and on the corresponding spectral gap estimates due to Holley, Kusuoka and Stroock.

Keywords: Stochastic algorithms, simulated annealing, homogenization, probability measures on compact Riemannian manifolds, intrinsic means, instantaneous invariant measures, Gibbs measures, spectral gap at small temperature.

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