Available online at www.sciencedirect.com

ScienceDirect

Stochastic Processes and their Applications 124 (2014) 3480–3495

stochastic processes and their applications

www.elsevier.com/locate/spa

New families of subordinators with explicit transition probability semigroup

J. Burridge[a], A. Kuznetsov[b], M. Kwaśnicki[c], A.E. Kyprianou[d,*]

[a] *Department of Mathematics, University of Portsmouth, Lion Gate Building, Lion Terrace, Portsmouth, Hampshire, PO1 3HF, UK*

[b] *Department of Mathematics and Statistics, York University, 4700 Keele Street, Toronto, Ontario, M3J 1P3, Canada*

[c] *Institute of Mathematics and Computer Science, Wroclaw University of Technology, ul. Wybrzeze Wyspiańskiego 27, 50-370 Wroclaw, Poland*

[d] *Department of Mathematical Sciences, University of Bath, Claverton Down, Bath, BA2 7AY, UK*

Received 5 February 2014; received in revised form 7 June 2014; accepted 9 June 2014
Available online 14 June 2014

Abstract

There exist only a few known examples of subordinators for which the transition probability density can be computed explicitly along side an expression for its Lévy measure and Laplace exponent. Such examples are useful in several areas of applied probability. For example, they are used in mathematical finance for modeling stochastic time change. They appear in combinatorial probability to construct sampling formulae, which in turn is related to a variety of issues in the theory of coalescence models. Moreover, they have also been extensively used in the potential analysis of subordinated Brownian motion in dimension $d \geq 2$. In this paper, we show that Kendall's classic identity for spectrally negative Lévy processes can be used to construct new families of subordinators with explicit transition probability semigroups. We describe the properties of these new subordinators and emphasize some interesting connections with explicit and previously unknown Laplace transform identities and with complete monotonicity properties of certain special functions.
© 2014 Published by Elsevier B.V.

MSC: 60G51; 44A10

Keywords: Subordinator; Kendall identity; Explicit transition density; Laplace transform identity; Bessel functions; Lambert W-function; Gamma function; Complete monotonicity; Generalized gamma convolutions

* Corresponding author. Tel.: +44 7837912198.
E-mail addresses: james.burridge@port.ac.uk (J. Burridge), kuznetsov@mathstat.yorku.ca (A. Kuznetsov), mateusz.kwasnicki@pwr.wroc.pl (M. Kwaśnicki), a.kyprianou@bath.ac.uk (A.E. Kyprianou).

http://dx.doi.org/10.1016/j.spa.2014.06.005
0304-4149/© 2014 Published by Elsevier B.V.

1. Introduction

Subordinators with explicit transition semigroups have proved to be objects of broad interest on account of their application in a variety of different fields. We highlight three of them here. The first case of interest occurs in mathematical finance, where subordinators are used to perform time-changes of other stochastic processes to model the effect of stochastic volatility in asset prices, see for example [5,7]. A second application occurs in the theory of potential analysis of subordinated Brownian motion in high dimensions, which has undergone significant improvements thanks to the study of a number of key examples, see for example [26,15]. A third area in which analytic details of the transition semigroup of a subordinator can lead to new innovations is that of combinatorial stochastic processes. A variety of sampling identities are intimately related to the range of particular subordinators, see for example [10]. Moreover this can also play an important role in the analysis of certain coalescent processes, see [22].

In this paper we will use a simple idea based on Kendall's identity for spectrally negative Lévy processes to construct some new families of subordinators with explicit transition semigroup. Moreover, we describe their properties, with particular focus on the associated Lévy measure and Laplace exponent in each of our new examples. The inspiration for the main idea in this paper came about by digging deeper into [4], where a remarkable identity appears in the analysis of the relationship between the first passage time of a random walk and the total progeny of a discrete-time, continuous-state branching process.

The rest of the paper is organized as follows. In the next section we remind the reader of Kendall's identity and thereafter, proceed to our main results. These results give a simple method for generating examples of subordinators with explicit transition semigroups as well as simultaneously gaining access to analytic features of their Lévy measure and Laplace exponent. In Section 3 we put our main results to use in generating completely new examples. Finally, in Section 4 we present some applications of these results to explicit Laplace transform identities and complete monotonicity properties of certain special functions.

2. Kendall's identity and main results

Let ξ be a spectrally negative Lévy process with Laplace exponent defined by

$$\psi(z) := \ln \mathbb{E}[\exp(z\xi_1)], \quad z \geq 0. \tag{1}$$

In general, the exponent ψ takes the form

$$\psi(z) = az + \frac{1}{2}\sigma^2 z^2 + \int_{(-\infty,0)} (e^{zx} - 1 - zx\mathbf{1}_{(x>-1)})\Pi_\xi(dx)$$

where $a \in \mathbb{R}$, $\sigma^2 \geq 0$ and Π_ξ is a measure concentrated on $(-\infty, 0)$ that satisfies $\int_{(-\infty,0)}(1 \wedge x^2)\Pi_\xi(dx) < \infty$, and is called the Lévy measure. From this definition, it is easy to deduce that ψ is convex on $[0, \infty)$, and it satisfies $\psi(0) = 0$ and $\psi(+\infty) = +\infty$. Hence, for every $q > 0$, there exists a unique solution $z = \phi(q) \in (0, \infty)$ to the equation $\psi(z) = q$. We will define $\phi(0) = \phi(0^+)$. Note that $\phi(0) = 0$ if and only if $\psi'(0) \geq 0$, which, by a simple differentiation of (1), is equivalent to $\mathbb{E}[\xi_1] \geq 0$.

Let us define the first passage times

$$\tau_x^+ := \inf\{t > 0 : \xi_t > x\}, \quad x \geq 0. \tag{2}$$

Download English Version:

https://daneshyari.com/en/article/10527202

Download Persian Version:

https://daneshyari.com/article/10527202

Daneshyari.com