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Greedy clearing of persistent Poissonian dust

Q1 L.T. Rolla*, V. Sidoravicius, L. Tournier

Instituto de Matemática Pura e Aplicada, Université Paris XIII, France

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Abstract

Given a Poisson point process on $\mathbb{R}$, assign either one or two marks to each point of this process, independently of the others. We study the motion of a particle that jumps deterministically from its current location to the nearest point of the Poisson point process which still contains at least one mark, and removes one mark per each visit. A point of the Poisson point process which is left with no marks is removed from the system. We prove that the presence of any positive density of double marks leads to the eventual removal of every Poissonian point.

Q2

0. Introduction

Given a Poisson point process $\mathcal{P} \subseteq \mathbb{R}$, we assign a random number $N_x \in \{1, 2, 3, \ldots\}$ of marks to each point $x \in \mathcal{P}$, independently from the values assigned to the other points in $\mathcal{P}$. Consider the following discrete motion of a particle. It starts at the origin $\mathbf{0}$, jumps to the nearest point $x \in \mathcal{P}$, and removes one mark from x. Then it carries this procedure indefinitely: always removing one mark at its current position, and targeting for its next jump the nearest $y \in \mathcal{P}$ still containing a mark.

If $\mathbb{P}(N_x = 1) = 1$, an elementary application of the Borel–Cantelli lemma shows that this motion eventually chooses a random direction, and drifts away, leaving a half-line unvisited. In this work we show that for $\mathbb{P}(N_x = 1) < 1$ and N_x taking values on $\{1, 2\}$, every mark and, as a consequence, every Poissonian point is eventually removed from $\mathbb{R}$. We conjecture that the same is true assuming only $\mathbb{P}(N_x = 1) < 1$.

* Corresponding author.
E-mail address: leorolla@impa.br (L.T. Rolla).

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Few words on the motivation and the background. A greedy algorithm reflects the strategy of maximizing the performance on the short run. Suppose that the task is to visit a given set of points within an infinite region, and the strategy is always to choose the nearest non-visited point. The model was introduced in [6], where it was called the "local traveling salesman problem" or the "tourist walk".[1] A tourist wishes to pay a visit to every landmark, and always goes to the nearest non-visited one. Does this strategy succeed? The answer may depend on the dimension.

If the landmarks form a Poisson point process on $\mathbb{R}$, the answer is simple. Consider the region spanning between the leftmost non-visited Poisson point on the right of the walk and the rightmost non-visited Poisson point on its left. The length of this region increases with each step of the walk, and a standard application of the Borel–Cantelli lemma implies that the walk crosses it only finitely many times. As a consequence, the walk eventually begins to move monotonically either to $+\infty$ or to $-\infty$, thus leaving infinitely many Poisson points not visited.

On $\mathbb{R}^d$, $d \geqslant 2$, it is less clear what one should expect from the behavior of the walk. This is due to certain self-repelling mechanism which lies in the nature of the process. To make this more explicit, we introduce the following *explorer*[2] process. The explorer starts at $S_0 = \mathbf{0}$. Sample an exponentially-distributed random variable A_1, which is interpreted as the volume that the particle is capable to explore at the first step. Let $D_1 \subseteq \mathbb{R}^2$ be the unique ball centered at S_0 and whose volume satisfies $|D_1| = A_1$. The new position S_1 of the explorer is then sampled uniformly on ∂D_1. For the second step, we sample a new exponentially-distributed random variable A_2, and let D_2 be the unique ball centered at S_1 such that $|D_2 \setminus D_1| = A_2$. The position S_2 is then sampled uniformly on $(\partial D_2) \setminus D_1$. In general, given positions $S_0, \ldots, S_{k-1}$, we sample a new exponentially-distributed random variable A_k, and let D_k be the unique ball centered at S_{k-1} such that

$$|D_k \setminus (D_1 \cup \cdots \cup D_{k-1})| = A_k, \tag{1}$$

and sample S_k uniformly on

$$(\partial D_k) \setminus (D_1 \cup \cdots \cup D_{k-1}). \tag{2}$$

The sequence S_n has the same law as the path performed by the tourist walk. Fig. 1 illustrates the first five steps of the explorer.

Note that if requirement (2) is replaced by a choice of S_k uniformly on ∂D_k, then one may view the walk as a Brownian motion observed at random times given by (1). Recurrence of the Brownian motion in the plane implies that every point is eventually covered by a disc D_k, while transience in higher dimensions implies that some regions are never explored. Requirement (2) above forces new positions of the walk to be chosen away from the previously explored region, which is responsible for a *local self-repulsion* of the process. In the Euclidean space $\mathbb{R}^d$ with $d \geqslant 3$, one naturally expects from the above description that some landmarks remain unvisited forever. In the plane however, the question is more delicate. Fig. 2 displays simulations of the process.

A surprising case is that of a strip $\mathbb{R} \times [0, \varepsilon]$, which turns out to be drastically different from the line. In numerical studies, a very intricate behavior, including heavy-tailed backtrack lengths, was

[1] The model is reminiscent of walks in rugged landscapes or zero-temperature spin-glass dynamics. It was advertised in [8] and studied in [7,3], and further discussed in [2].

[2] We call this process the explorer process due to its close analogy with the rancher process, introduced and studied in [1], and also in [9].

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