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Asymptotics for functionals of self-normalized residuals of discretely observed stochastic processes

Hiroki Masuda

Institute of Mathematics for Industry, Kyushu University, 744 Motooka, Nishi-ku, Fukuoka 819-0395, Japan

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Abstract

The purpose of this paper is to derive the stochastic expansion of self-normalized-residual functionals stemming from a class of diffusion type processes observed at high frequency, where total observing period may or may not tend to infinity. The result enables us to construct some explicit statistics for goodness of fit tests, consistent against "presence of a jump component" and "diffusion-coefficient misspecification"; then, the acceptance of the null hypothesis may serve as a collateral evidence for using the correctly specified diffusion type model. Especially, our asymptotic result clarifies how to remove the bias caused by plugging in a diffusion-coefficient estimator.

MSC: 62F05; 62M07; 62M09

Keywords: Goodness of fit; High-frequency data; Self-normalized residuals; Stochastic differential equation

1. Introduction

Statistical model diagnostics are often done through suitable residual statistics. What we address in this paper is to derive a stochastic expansion of self-normalized-residual functionals related to a class of diffusion type models observed at high frequency. Our main result enables us to perform handy goodness of fit tests.

We begin with background of the present study, with mentioning some previous related works. Consider the diffusion process

$$dX_t = a(X_t, \alpha)dt + b(X_t, \beta)dw_t, \tag{1}$$

E-mail address: hiroki@imi.kyushu-u.ac.jp.

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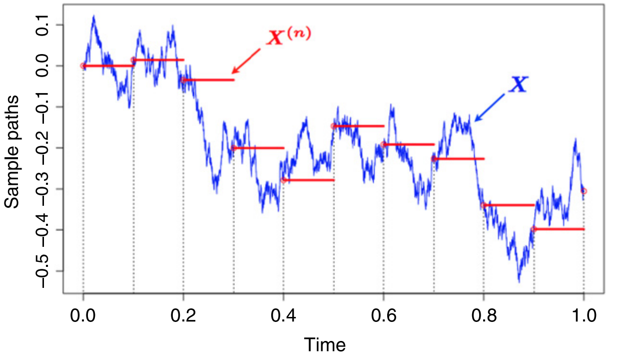

Fig. 1. Sample-path plots of a stochastic process X and its Euler approximation $X^{(n)}$.

where the coefficients (a, b) is supposed to be known except for the finite-dimensional parameter $\theta := (\alpha, \beta) \in \Theta \subset \mathbb{R}^p$, based on a discrete-time sample $(X_0, X_{t_1^n}, X_{t_2^n}, \dots, X_{t_n^n})$ with $t_i^n = ih_n$ for the sampling step size $h_n \to 0$. What we really have over the interval $[0, T_n]$, where $T_n := t_n^n$, is just an *Euler–Maruyama approximation process*

$$X_t^{(n)} := X_{h_n[t/h_n]}. \tag{2}$$

In principle, the process $X^{(n)}$ can be defined for any process X, and sample paths of X^n are piecewise constant and right-continuous (Fig. 1). A fundamental statistical problem is to estimate θ based on the sample $(X_t^{(n)} : t \in [0, T_n])$, for which we know several results.

Suppose that $nh_n^2 \to 0$ and that there exists a true value of $\theta_0 \in \Theta$. Then there exist several results concerning how to construct an $\sqrt{n}$-*consistent* estimator of θ_0. The most important one is the Gaussian Quasi-Maximum Likelihood Estimator (GQMLE) defined to be any $\hat{\theta}_n = (\hat{\alpha}_n, \hat{\beta}_n) \in \operatorname{argmax}\mathbb{M}_n^\star$ where

$$\mathbb{M}_n^\star(\theta) := \sum_{i=1}^n \log\phi\left(X_{t_i^n}; X_{t_{i-1}^n} + h_n a(X_{t_{i-1}^n}, \alpha), h_n b^{\otimes 2}(X_{t_{i-1}^n}, \beta)\right),$$

with $\phi(\cdot; \mu, \Sigma)$ denoting the Gaussian density with mean vector μ and covariance matrix Σ, and $b^{\otimes 2} := bb^\top$ where $\top$ denotes the transpose. The form of $\mathbb{M}_n^\star$ comes from, based on the Euler approximation (2), the local Gaussian approximation of the conditional distributions under the true measure:

$$\mathcal{L}\left(X_{ih_n} | X_{(i-1)h_n} = x\right) \approx \mathcal{N}\left(x + h_n a(x, \alpha_0), h_n b^{\otimes 2}(x, \beta_0)\right). \tag{3}$$

Under the ergodicity of X as well as regularity conditions on (a, b), the GQMLE can be asymptotically normal and efficient with the optimal rates (of regular estimators) $\sqrt{nh_n}$ and $\sqrt{n}$ for α and β, respectively (cf. [8,14], and also [27] as well as the references therein); this means that we can estimate the true value of β more quickly than that of α. Under weaker assumption $nh_n^p \to 0$ for some $p \geq 2$, [27] proposed some adaptive estimation procedures, which exhibits computational stability in optimizing the quasi-likelihood function $\mathbb{M}_n^\star$, and derived the convergence of moments in addition to the asymptotic normality; the optimal rate in estimating (α, β) is the same as above; see also [28,29]. Furthermore, the GQMLE admits a