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Invariance principles for generalized domains of semistable attraction

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Abstract

Let $X, X_1, X_2, \ldots$ be independent and identically distributed $\mathbb{R}^d$-valued random vectors and assume X belongs to the generalized domain of attraction of some operator semistable law without normal component. Then without changing its distribution, one can redefine the sequence on a new probability space such that the properly affine normalized partial sums converge in probability and consequently even in L^p (for some $p > 0$) to the corresponding operator semistable Lévy motion.

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1. Introduction

Let $\mathbb{R}^d$ be the space of d-dimensional real column vectors endowed with the Euclidean norm $\|\cdot\|$. Let $X, X_1, X_2, \ldots$ be independent and identically distributed (i.i.d.) $\mathbb{R}^d$-valued random vectors and assume that X belongs to the *generalized domain of operator semistable attraction* (abbreviated as GDOSA) of some random vector Y. This means that for some increasing sequence $(k(n))$ of natural numbers tending to infinity, with $k(n+1)/k(n) \to \bar{c} \geq 1$ as $n \to \infty$, nonrandom shifts $a(n) \in \mathbb{R}^d$ and linear operators $A(n)$, the affine normalized partial sum converges in distribution to Y, i.e.

$$A(n)(X_1 + \cdots + X_{k(n)}) + a(n) \Rightarrow Y \quad \text{as } n \to \infty. \tag{1.1}$$

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The limit law $\nu = \mathcal{L}(Y)$ is called *operator semistable*. In the excellent paper Jajte [7] introduced and characterized the notion of an operator semistable law in $\mathbb{R}^d$.

We say that X belongs to the strict GDOSA if (1.1) holds with $a(n) = 0$. When $k(n) \equiv n$ in (1.1), we say that X belongs to the *generalized domain of operator stable attraction*. Furthermore, if $A(n)$ is a nonrandom real number then we say that X belongs to the *domain of attraction of a stable law*.

We say that Y is *full*, meaning that the support of the distribution $\mathcal{L}(Y)$ is not contained in any $(d-1)$-dimensional hyperplane of $\mathbb{R}^d$. This forces $A(n)$ to be invertible for all large n and hence we can assume without loss of generality that $A(n)$ is invertible for all n. It is well-known (see, e.g., [7,2,12]) that a full probability measure ν is an operator semistable measure if and only if ν is infinitely divisible and there exist a number $c > 1$, a vector $b \in \mathbb{R}^d$ and a linear operator E (an invertible $d \times d$ matrix) such that

$$\nu^c = c^E \nu * \delta(b) \tag{1.2}$$

holds. Then ν is called (c^E, c) operator semistable. Here $\mu * \nu$ denotes the convolution of two measures μ and ν, ν^c is the c-fold convolution power of the infinitely divisible law ν, $c^E = \exp(E \log c)$ where

$$\exp(E) = \sum_{j=0}^{\infty} \frac{E^j}{j!}$$

is the usual exponential operator with the convention $E^0 = I$ the identity $d \times d$ matrix and $\delta(b)$ denotes the point mass in a $b \in \mathbb{R}^d$. We write $X \in \mathrm{GDOSA}(Y, c)$ if (1.2) holds. By Jajte [7], the distribution $\nu = \mathcal{L}(Y)$ in (1.1) is *operator stable* if $\bar{c} = 1$ and (1.2) holds for all $c > 1$. This means that operator semistable laws include operator stable laws as special cases. In fact, following the argumentation in [15], the distribution $\nu = \mathcal{L}(Y)$ is the most general setting in which useful information about the distribution of X can be obtained from the limit relation (1.1). By the theorem in [2], $c \in \{\bar{c}, \bar{c}^2\}$ in (1.2) if $\bar{c} > 1$. We call ν stable with index α $(0 < \alpha < 2)$ or multivariable stable, if (1.2) holds for the exponent $E = (1/\alpha)I$.

The limit law $\nu = \mathcal{L}(Y)$ in (1.1) is infinitely divisible and hence representable by the Lévy–Khinchin triple $[a, Q, \phi]$, where $a \in \mathbb{R}^d$, $Q(t)$ is a nonnegative definite quadratic form on $\mathbb{R}^d$ and ϕ is a Lévy measure, that is, the characteristic function of ν is $\exp(\psi(t))$ with

$$\psi(t) = i\langle a, t\rangle - \frac{1}{2}Q(t) + \int_{\mathbb{R}^d \setminus \{0\}} \left(e^{i\langle t, x\rangle} - 1 - \frac{i\langle t, x\rangle}{1 + \|x\|^2} \right) \phi(dx)$$

for all $t \in \mathbb{R}^d$, where $\langle \cdot, \cdot \rangle$ denotes the standard inner product on $\mathbb{R}^d$. The triple $[a, Q, \phi]$ is unique. It follows from Hudson et al. [6] and Scheffler [19] that if $\phi \neq 0$ then $\mathbb{E}\|X\|^2 = \infty$ and hence X is heavy tailed unless ν is a purely Gaussian law. For more information on the infinitely divisible distributions one can refer to Sato [18].

The purpose of this paper is to investigate weak and L^p-invariance principles for heavy-tailed random vectors and hence we assume that ν is nonnormal, i.e. $Q = 0$.

Scheffler [20] investigated the large deviation results and various variants of Chover's laws of the iterated logarithm (LIL) for distributions belonging to the strict GDOSA of a nonnormal operator semistable law. Scheffler and Becker-Kern [24] investigated the LILs for randomly stopped sums of random vectors belonging to the generalized domain of semistable attraction of some nonnormal law. Meerschaert and Scheffler [13] studied the invariance principle in