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Backward stochastic differential equations associated to jump Markov processes and applications

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Abstract

In this paper we study backward stochastic differential equations (BSDEs) driven by the compensated random measure associated to a given pure jump Markov process X on a general state space K. We apply these results to prove well-posedness of a class of nonlinear parabolic differential equations on K, that generalize the Kolmogorov equation of X. Finally we formulate and solve optimal control problems for Markov jump processes, relating the value function and the optimal control law to an appropriate BSDE that also allows to construct probabilistically the unique solution to the Hamilton–Jacobi–Bellman equation and to identify it with the value function.
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1. Introduction

In this paper we introduce and solve a class of backward stochastic differential equations (BSDEs for short) driven by a random measure associated to a given jump Markov process. We apply the results to study nonlinear variants of the Kolmogorov equation of the Markov process and to solve optimal control problems.

Let us briefly describe our framework. Our starting point is a pure jump Markov process X on a general state space K. It is constructed in a usual way starting from a positive measure

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$A \mapsto \nu(t, x, A)$ on K, depending on $t \geq 0$ and $x \in K$ and called rate measure, that specifies the jump rate function $\lambda(t, x) = \nu(t, x, K)$ and the jump measure $\pi(t, x, A) = \nu(t, x, A)/\lambda(t, x)$. If the process starts at time t from x then the distribution of its first jump time T_1 is described by the formula

$$\mathbb{P}(T_1 > s) = \exp\left(-\int_t^s \lambda(r, x)\, dr\right), \tag{1.1}$$

and the conditional probability that the process is in A immediately after a jump at time $T_1 = s$ is

$$\mathbb{P}(X_{T_1} \in A \mid T_1 = s) = \pi(s, x, A),$$

see below for precise statements. We denote by $\mathbb{F}$ the natural filtration of the process X. Denoting by T_n the jump times of X, we consider the marked point process (T_n, X_{T_n}) and the associated random measure $p(dt\, dy) = \sum_n \delta_{(T_n, X_{T_n})}$ on $(0, \infty) \times K$, where δ denotes the Dirac measure. In the Markovian case the dual predictable projection $\tilde{p}$ of p (shortly, the compensator) has the following explicit expression

$$\tilde{p}(dt\, dy) = \nu(t, X_{t-}, dy)\, dt.$$

In the first part of the paper we introduce a class of BSDEs driven by the compensated random measure $q(dt\, dy) := p(dt\, dy) - \tilde{p}(dt\, dy)$ and having the following form

$$Y_t + \int_t^T \int_K Z_r(y)\, q(dr\, dy) = g(X_T) + \int_t^T f(r, X_r, Y_r, Z_r(\cdot))\, dr, \quad t \in [0, T], \tag{1.2}$$

for given generator f and terminal condition g. Here Y is real-valued, while Z is indexed by $y \in K$, i.e. it is a random field on K, with appropriate measurability conditions, and the generator depends on Z in a general functional way. Relying upon the representation theorem for the $\mathbb{F}$-martingales by means of stochastic integrals with respect to q we can prove several results on (1.2), including existence, uniqueness and continuous dependence on the data.

In spite of the large literature devoted to random measures (or equivalently to marked point processes) there are relatively few results on their connections with BSDEs. General nonlinear BSDEs driven by the Wiener process were first solved in [18]. Since then, many generalizations have been considered where the Wiener process was replaced by more general processes. Backward equations driven by random measures have been studied in [21,2,20,17], in view of various applications including stochastic maximum principle, partial differential equations of nonlocal type, quasi-variational inequalities and impulse control. The stochastic equations addressed in these papers are driven by a Wiener process and by a jump process, but the latter is only considered in the Poisson case. More general results on BSDEs driven by random measures can be found in the paper [22], but they require a more involved formulation; moreover, in contrast to [21] or [2], the generator f depends on the process Z in a specific way (namely as an integral of a Nemytskii operator) that prevents some of applications that we wish to address, for instance optimal control problems.

In this paper X is not defined as a solution of a stochastic equation, but rather constructed as described above. While we limit ourselves to the case of a pure jump process X, we can allow great generality. Roughly speaking, we can treat all strong Markov jump processes such that the distribution of holding times admits a rate function $\lambda(t, x)$ as in (1.1): compare Remark 2.1-3. The process X is not required to be time-homogeneous, the holding times are not necessarily exponentially distributed and can be infinite with positive probability. Our main restriction is that

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