Available online at www.sciencedirect.com

ScienceDirect

Stochastic Processes and their Applications 124 (2014) 373–384

stochastic processes and their applications

www.elsevier.com/locate/spa

On the characterisation of honest times that avoid all stopping times

Constantinos Kardaras

Statistics Department, London School of Economics and Political Science, 10 Houghton Street, London, WC2A 2AE, UK

Received 30 October 2012; received in revised form 22 July 2013; accepted 30 July 2013
Available online 21 August 2013

Abstract

We present a short and self-contained proof of the following result: a random time is an honest time that avoids all stopping times if and only if it coincides with the (last) time of maximum of a nonnegative local martingale with zero terminal value and no jumps while at its running supremum, where the latter running supremum process is continuous. Illustrative examples involving local martingales with discontinuous paths are provided.
© 2013 Elsevier B.V. All rights reserved.

MSC: 60G07; 60G44

Keywords: Honest times; Times of maximum; Non-negative local martingales; Running supremum

1. The characterisation result

1.1. Honest times that avoid all stopping times

Let $(\Omega, \mathbf{F}, \mathbb{P})$ be a filtered probability space, where $\mathbf{F} = (\mathcal{F}_t)_{t \in \mathbb{R}_+}$ is a filtration satisfying the usual conditions of right-continuity and saturation by $\mathbb{P}$-null sets of $\mathcal{F} := \bigvee_{t \in \mathbb{R}_+} \mathcal{F}_t$. All (local) martingales and supermartingales on $(\Omega, \mathbf{F}, \mathbb{P})$ are assumed to have $\mathbb{P}$-a.s. càdlàg paths.

Definition 1.1. A *random time* is a $[0, \infty]$-valued, $\mathcal{F}$-measurable random variable. The random time ρ is said to *avoid all stopping times* if $\mathbb{P}[\rho = \tau] = 0$ holds whenever τ is a (possibly, infinite-valued) stopping time. The random time ρ is called an *honest time* if for all $t \in \mathbb{R}_+$ there exists an $\mathcal{F}_t$-measurable random variable R_t such that $\rho = R_t$ holds on $\{\rho \leq t\}$.

E-mail addresses: k.kardaras@lse.ac.uk, langostas@gmail.com.

0304-4149/$ - see front matter © 2013 Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.spa.2013.07.012

Honest times constitute the most important class of random times outside the realm of stopping times. They have been extensively studied in the literature, especially in relation to filtration enlargements. It is impossible to present here the vast literature on the subject of honest times; we indicatively mention the early papers [1–3,9,15], as well as the monographs [8,10]. Lately, there has been considerable revival to the study of honest times, due to questions arising from the field of Financial Mathematics—see, for example, [5,11,6] and the references therein.

1.2. The class $\mathcal{M}_0$

Define $\mathcal{M}_0$ to be the class of all nonnegative local martingales L such that $L_0 = 1$, the running supremum process $L^* := \sup_{t \in [0,\cdot]} L_t$ is continuous (up to a $\mathbb{P}$-evanescent set), and $\mathbb{P}[L_\infty = 0] = 1$ holds, where $L_\infty := \lim_{t \to \infty} L_t$. (Note that the limit in the definition of L_∞ exists in the $\mathbb{P}$-a.s. sense, in view of the nonnegative supermartingale convergence theorem.)

For $L \in \mathcal{M}_0$, define[1]

$$\rho_L := \sup \left\{ t \in \mathbb{R}_+ \mid L_{t-} = L^*_{t-} \right\}, \tag{1.1}$$

where note that $L_{0-} = 1 = L^*_{0-}$ implies that the (random) set $\left\{ t \in \mathbb{R}_+ \mid L_{t-} = L^*_{t-} \right\}$ is non-empty. Since $\mathbb{P}[L_\infty = 0] = 1$ holds for $L \in \mathcal{M}_0$, it follows that $\mathbb{P}[\rho_L < \infty] = 1$.

For $L \in \mathcal{M}_0$ and $t \in \mathbb{R}_+$, define $R_t := \sup \left\{ s \in [0,t] \mid L_{s-} = L^*_{s-} \right\} \wedge t$, which is an $\mathcal{F}_t$-measurable random variable such that $\rho_L = R_t$ holds on $\{\rho_L \le t\}$. It follows that ρ_L is an honest time whenever $L \in \mathcal{M}_0$.

1.3. The class $\mathcal{L}_0$

Let $L \in \mathcal{M}_0$. In view of (1.1), ρ_L coincides with the end of the predictable set $\left\{ L_- = L^*_- \right\}$. Using the $\mathbb{P}$-a.s. left-continuity of L_- and the $\mathbb{P}$-a.s. continuity of L^*, as well as the definition of ρ_L from (1.1), we obtain that $L_{\rho_L -} = L^*_{\rho_L -} = L^*_{\rho_L}$ holds in the $\mathbb{P}$-a.s. sense. (In particular, the "sup" in (1.1) is really a "max".) If one wishes to ensure that ρ_L is an actual time of maximum of L, it suffices to ask that L has no jumps when L_- is at its running supremum. Motivated by this observation, we define the class $\mathcal{L}_0$ to consist of all $L \in \mathcal{M}_0$ with the additional property that $\left\{ L_- = L^*_- \right\} \subseteq \{\Delta L = 0\}$ holds up to a $\mathbb{P}$-evanescent set. Whenever $L \in \mathcal{L}_0$, it $\mathbb{P}$-a.s. holds that $L_{\rho_L -} = L_{\rho_L} = L^*_{\rho_L}$; in fact, as Theorem 1.2 will imply, the previous random variables are also equal to L^*_∞, which makes ρ_L a time of *overall* maximum of $L \in \mathcal{L}_0$. On the other hand, if $L \in \mathcal{M}_0 \setminus \mathcal{L}_0$ it may happen that L does not achieve its overall supremum; furthermore, it may also happen that ρ_L fails to avoid all stopping times—for both previous points, see Remark 1.4.

1.4. The characterisation result

The following result shows that, for $L \in \mathcal{L}_0$, the random time ρ_L defined in (1.1) is the canonical example of an honest time that avoids all stopping times.

Theorem 1.2. *For a random time ρ, the following two statements are equivalent.*

(1) *ρ is an honest time that avoids all stopping times.*

(2) *$\rho = \rho_L$ holds in the $\mathbb{P}$-a.s. sense for some $L \in \mathcal{L}_0$.*

Under (any of) the previous conditions, the equality $L_{\rho-} = L_\rho = L^*_\infty$ holds in the $\mathbb{P}$-a.s. sense; furthermore, $\mathbb{P}[\rho > t \mid \mathcal{F}_t] = L_t / L^*_t$ in the $\mathbb{P}$-a.s. sense is valid for all $t \in \mathbb{R}_+$.

[1] As usual, for any càdlàg process X, X_- denotes the càglàd process defined in a way such that X_{t-} is the left limit of X at $t \in (0, \infty)$; by convention, we also set $X_{0-} = X_0$.

Download English Version:

https://daneshyari.com/en/article/10527239

Download Persian Version:

https://daneshyari.com/article/10527239

Daneshyari.com